MIFP-09-26 USTC-ICTS-09-09

Most General Spherically Symmetric M2-branes and Type IIB Strings

Zhao-Long Wang [†] and H. Lü [⋆]

[†] *Interdisciplinary Center for Theoretical Study,*
University of Science and Technology of China, Hefei, Anhui 230026, China

[⋆] *China Institute for Advanced Study,*
China Economics and Management Academy,
Central University of Finance and Economics, Beijing, 100081, China

ABSTRACT

We obtain the most general spherically symmetric M2-branes and type IIB strings, with $\mathbb{R}^{1,2} \times SO(8)$ and $\mathbb{R}^{1,1} \times SO(8)$ isometries, respectively. We find that there are 12 different classes of M2-branes, and we study their curvature properties. In particular, we obtain new smooth M2-brane wormholes that connect two asymptotic regions: one is flat and the other can be either flat or $\mathrm{AdS}_4 \times S^7$. We find that these wormholes are traversable with certain timelike trajectories. We also obtain the most general Ricci-flat solutions in five dimensions with $\mathbb{R}^{1,1} \times SO(3)$ isometries.

Contents

1 Introduction

The higher dimensionality and the inclusion of higher form fields in string and M theories allow a variety of p-brane solitons in the spectrum [1]. These objects are generalizations of charged black holes and have been playing an important role in understanding the non-perturbative aspects of the string theories. A p-brane soliton is defined to be a spacetime configuration that splits the D-dimensional spacetime into the world volume of $(p + 1)$ dimensions and the transverse space of $(D - p - 1)$ dimensions. The electrically charged p-brane is supported by a $(p + 1)$-form potential, while the magnetically charged one is by a $(D - p - 1)$-form. The simplest type of p-branes has the Minkowski world volume and spherically symmetric transverse space. A classification of such p-branes in maximal supergravities was given in [2]. Lower-dimensional p-branes can be lifted to higher dimensions and become intersecting M2-branes or type II strings [3, 4]. Classifications of brane intersections in maximal supergravities were given in [5, 6].

In this paper, we consider a more general class of p-brane solutions, focusing on M2-branes and type IIB strings. While we still retain the spherical symmetry in the trans-

verse space for simplicity, we shall relax the condition for the world volume from being Minkowskian to having instead the $\mathbb{R}^{1,p}$ isometry. A simple example is a p-brane with gravitational a pp-wave propagating in the world volume. Such a p-brane has the isometry of $\mathbb{R}^{1,p} \times SO(D-p-1)$. The metric takes the form

$$ds_D^2 = r^2 d\Omega_{D-p-2}^2 + \frac{dr^2}{f} + \sum_{\mu,\nu=0}^{p} g_{\mu\nu} dz^\mu dz^\nu \,, \tag{1.1}$$

where f and $g_{\mu\nu}$ depend only on the radial variable r of the transverse space. The world volume coordinates are z^μ, with $z^0 = t$ denoting the time direction. The way to obtain such a solution is to observe that one can perform Kaluza-Klein reduction on the coordinates z^μ, on which no field depends. The resulting solution in $(D-p-1)$ dimensions is an instanton supported by a certain scalar coset G/H. In the case of pure gravity, the scalar coset is $\frac{GL(p+1,\mathbb{R})}{SO(1,p)}$, for which the most general solution was obtained in [7]. (See also [8, 9].)

In section 2, we consider the reduction of M2-branes on the world volume. The Kaluza-Klein reduction of eleven-dimensional supergravity on $\mathbb{R}^{1,2}$ gives rise to a scalar coset of $\frac{SL(3,\mathbb{R})}{SO(1,2)} \times \frac{SL(2,\mathbb{R})}{SO(1,1)}$ [10, 11]. This enables us to make use of the results in [7] and obtain the most general M2-brane solutions. We find that there are twelve classes of solutions, including the previously known extremal [12] or nonextremal M2-branes [13, 14]. We also find new smooth M2-brane wormhole solutions. We study the curvature properties of these solutions. In particular, in section 3, we focus on the wormholes and demonstrate that they are not traversable geodesically but traversable with certain timelike trajectories, as in the case of the previously known Ricci-flat wormholes [15, 16, 17, 7].

The $\frac{SL(3,\mathbb{R})}{SO(1,2)} \times \frac{SL(2,\mathbb{R})}{SO(1,1)}$ scalar coset can also be obtained from the reduction of type IIB supergravity on $\mathbb{R}^{1,1}$. This enables us to lift the instanton solutions back to $D = 10$ and obtain the most general type IIB string solutions. We do this in section 4. It is worth pointing out that, in the case of M theory on $\mathbb{R}^{1,2}$, the $SL(3,\mathbb{R})$ global symmetry is part of the general coordinate transformation, and hence we can use it to mod out the equivalent solutions related by coordinate transformation. In the case of type IIB supergravity on $\mathbb{R}^{1,1}$, it is the $SL(2,\mathbb{R})$ factor that is part of the general coordinate transformation. Thus the same instanton solution in $D = 8$ can lead to very different lifting to M2-branes or type IIB strings.

In section 5, we further consider five-dimensional pure gravity on $\mathbb{R}^{1,1}$, which becomes the scalar coset of $\frac{SL(3,\mathbb{R})}{SO(1,2)}$. This enables us to obtain the most general Ricci-flat solutions in five dimensions with $\mathbb{R}^{1,1} \times SO(3)$ isometry.

We conclude the paper in section 6.

2 General Spherically symmetric M2-branes

The world volume of M2-branes has three dimensions, including one time and two spatial directions. Performing Kaluza-Klein reduction on the world volume gives rise to instanton solutions in $D = 8$ Euclidean maximal supergravity. In this section, we review the scalar sector in $D = 8$ and obtain the most general spherically symmetric instanton solutions supported by the scalars. Lifting the solutions back to $D = 11$ gives rise to the most general spherically symmetric M2-branes, with $\mathbb{R}^{1,2} \times SO(8)$ isometries. We then study the curvature properties of these solutions.

2.1 $D = 8$ Euclidean maximal supergravity: the scalar sector

Eight-dimensional Euclidean maximal supergravity obtained from $\mathbb{R}^{1,2}$ reduction of eleven-dimensional supergravity was discussed in [10, 11]. Here we shall focus on the scalar sector. The bosonic sector of eleven-dimensional supergravity consists of the metric and a 3-form tensor field $A_{(3)}$. The action is given by [18]

$$S_{11} = \int d^{11}x\sqrt{-G}\left(R - \frac{1}{48}F_{(4)}^2\right) - \frac{1}{6}\int A_{(3)} \wedge F_{(4)} \wedge F_{(4)}\,, \tag{2.1}$$

where $F_{(4)} = dA_{(3)}$. There are a total of seven scalars in $D = 8$, six of which come from the metric, whose reduction ansatz is given by

$$\begin{aligned} ds_{11}^2 &= e^{-\frac{1}{3}\Phi} ds_8^2 + e^{\frac{2}{3}\Phi} dz^{\mathrm{T}} \mathcal{M}\, dz \\ &= e^{-\frac{1}{3}\Phi} ds_8^2 + e^{\frac{2}{3}\Phi}\Big(e^{\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2}(dz_1 + \chi_{12} dz_2 + \chi_{10} dt)^2 \\ &\qquad\qquad + e^{-\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2}(dz_2 + \chi_{20} dt)^2 - e^{-\frac{2}{\sqrt{3}}\Phi_2} dt^2\Big)\,, \end{aligned} \tag{2.2}$$

where $z = (z_1, z_2, t)$ and the matrix $\mathcal{M}$ in the metric involves five scalars, parameterizing the $\frac{SL(3,\mathbb{R})}{SO(1,2)}$ scalar coset, namely,

$$\mathcal{M} = \begin{pmatrix} e^{\Phi_1 + \frac{\Phi_2}{\sqrt{3}}}\chi_{10}^2 - e^{-\frac{2\Phi_2}{\sqrt{3}}} + e^{-\Phi_1 + \frac{\Phi_2}{\sqrt{3}}}\chi_{20}^2 & e^{\Phi_1 + \frac{\Phi_2}{\sqrt{3}}}\chi_{10} & e^{\Phi_1 + \frac{\Phi_2}{\sqrt{3}}}\chi_{12}\chi_{10} + e^{-\Phi_1 + \frac{\Phi_2}{\sqrt{3}}}\chi_{20} \\ e^{\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2}\chi_{10} & e^{\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2} & e^{\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2}\chi_{12} \\ e^{\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2}\chi_{12}\chi_{10} + e^{-\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2}\chi_{20} & e^{\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2}\chi_{12} & e^{-\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2} + e^{\Phi_1 + \frac{1}{\sqrt{3}}\Phi_2}\chi_{12}^2 \end{pmatrix}. \tag{2.3}$$

The parametrization of such a scalar coset was discussed in detail in [10, 7]. The reduction of the 3-form tensor field $A_{(3)}$ on $\mathbb{R}^{1,2}$ gives rise to one axionic scalar χ, given by

$$A_{(3)} = \chi\, dt \wedge dz_1 \wedge dz_2\,. \tag{2.4}$$

This axion and the breathing mode Φ in the metric form an $\frac{SL(2,\mathbb{R})}{SO(1,1)}$ scalar coset with

$$\tilde{\mathcal{M}} = \begin{pmatrix} -e^{\Phi} + e^{-\Phi}\chi^2 & e^{-\Phi}\chi \\ e^{-\Phi}\chi & e^{-\Phi} \end{pmatrix}. \tag{2.5}$$

The Lagrangian of the scalar sector, coupled to gravity, of Euclidean maximal supergravity in $D = 8$ is then given by

$$\begin{aligned} \mathcal{L}_8 &= \sqrt{g}\Big(R + \tfrac{1}{4}\mathrm{tr}(\partial_\mu \mathcal{M}^{-1}\partial^\mu \mathcal{M}) + \tfrac{1}{4}\mathrm{tr}(\partial_\mu \tilde{\mathcal{M}}^{-1}\partial^\mu \tilde{\mathcal{M}})\Big) \\ &= \sqrt{g}\Big(R - \tfrac{1}{2}(\partial\Phi)^2 + \tfrac{1}{2}e^{-2\Phi}(\partial\chi)^2 - \tfrac{1}{2}(\partial\Phi_1)^2 - \tfrac{1}{2}(\partial\Phi_2)^2 \\ &\qquad - \tfrac{1}{2}e^{2\Phi_1}(\partial\chi_{12})^2 + \tfrac{1}{2}e^{\Phi_1+\sqrt{3}\Phi_2}(\partial\chi_{10} - \chi_{20}\partial\chi_{12})^2 + \tfrac{1}{2}e^{-\Phi_1+\sqrt{3}\Phi_2}(\partial\chi_{20})^2\Big). \end{aligned} \tag{2.6}$$

2.2 Spherically symmetric solution

We now consider spherically symmetric instanton solutions to the Lagrangian (2.6) in eight dimensions. The metric ansatz is given by

$$ds_8^2 = \frac{1}{f(r)}dr^2 + r^2 d\Omega_7^2\,, \tag{2.7}$$

with all of the scalars depending only on the radial coordinate r. The equations of motion of such an ansatz for general scalar cosets were discussed in [7]. The Einstein equations in the directions of the foliating sphere S^7 imply that

$$12(1-f) - rf' = 0\,, \tag{2.8}$$

where a prime denotes a derivative with respect to r. Thus we have

$$f = 1 - \frac{a}{r^{12}}\,. \tag{2.9}$$

The Einstein equation associated with R_{rr} implies that

$$-\frac{7f'}{2r\,f} + \tfrac{1}{4}\mathrm{tr}(\mathcal{M}^{-1'}\mathcal{M}') + \tfrac{1}{4}\mathrm{tr}(\tilde{\mathcal{M}}^{-1'}\tilde{\mathcal{M}}') = 0\,. \tag{2.10}$$

The scalar equations of motion are given by

$$(\mathcal{M}^{-1}\dot{\mathcal{M}})^{\cdot} = 0\,, \qquad (\tilde{\mathcal{M}}^{-1}\dot{\tilde{\mathcal{M}}})^{\cdot} = 0 \tag{2.11}$$

where a dot denotes a derivative with respect to ρ, defined by

$$d\rho = \frac{dr}{r^7\sqrt{f}}\,. \tag{2.12}$$

Thus, we have

$$\rho = -\frac{1}{6\sqrt{a}} \arcsin\left(\frac{\sqrt{a}}{r^6}\right). \tag{2.13}$$

The second-order differential equations (2.11) for the scalars can be easily integrated to give rise to a set of first-order equations, given by

$$\mathcal{M}^{-1}\dot{\mathcal{M}} = \mathcal{C}\,, \qquad \tilde{\mathcal{M}}^{-1}\dot{\tilde{\mathcal{M}}} = \tilde{\mathcal{C}}\,, \tag{2.14}$$

where $\mathcal{C}$ and $\tilde{\mathcal{C}}$ are Lie-algebra valued constant matrices. Both $\mathcal{C}$ and $\tilde{\mathcal{C}}$ are traceless since $\det\mathcal{M} = \det\tilde{\mathcal{M}} = -1$. Substituting this and (2.9) into (2.10), we have the following constraint on these constant matrices, namely

$$\mathcal{I} \equiv -\tfrac{1}{2}\mathrm{tr}(\mathcal{C}^2) - \tfrac{1}{2}\mathrm{tr}(\tilde{\mathcal{C}}^2) = 2(D-1)(D-2)a = 84a\,. \tag{2.15}$$

Thus we see that the solutions are completely determined by the constant matrices $\mathcal{C}$ and $\tilde{\mathcal{C}}$. Such matrices for the $SL(n,\mathbb{R})/SO(1,n-1)$ scalar cosets were classified in [7, 8] that would yield different classes of solutions. If we turn off the scalars in $\tilde{\mathcal{M}}$, the solutions are then Ricci-flat, and fully classified in [7], including new smooth wormholes and new tachyon waves. If instead we turn off the scalars in $\mathcal{M}$, we may obtain supersymmetric [12] and non-supersymmetric M2-branes as in [19, 20].

We now consider the classifications of $\mathcal{C}$ and $\tilde{\mathcal{C}}$ and hence $\mathcal{M}$ and $\tilde{\mathcal{M}}$ in detail. We begin with $\mathcal{C}$ and $\mathcal{M}$, which were discussed in [7]. Here we shall just present the results.

Class I: The constant matrix $\mathcal{C}$ has a pair of complex eigenvalues. It is isomorphic to

$$\mathcal{C} = \begin{pmatrix} \alpha & -\beta & 0 \\ \beta & \alpha & 0 \\ 0 & 0 & -2\alpha \end{pmatrix}. \tag{2.16}$$

It follows that

$$\mathcal{M} = \begin{pmatrix} -e^{\alpha\rho}\cos\beta\rho & e^{\alpha\rho}\sin\beta\rho & 0 \\ e^{\alpha\rho}\sin\beta\rho & e^{\alpha\rho}\cos\beta\rho & 0 \\ 0 & 0 & e^{-2\alpha\rho} \end{pmatrix}. \tag{2.17}$$

Class II: The matrix $\mathcal{C}$ has three real eigenvalues, with one timelike and two spacelike eigenvectors. It is isomorphic to

$$\mathcal{C} = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & -(\alpha_1+\alpha_2) \end{pmatrix}. \tag{2.18}$$

Thus we have

$$\mathcal{M}=\begin{pmatrix}-e^{\alpha_1\rho} & 0 & 0\\ 0 & e^{\alpha_2\rho} & 0\\ 0 & 0 & e^{-(\alpha_1+\alpha_2)\rho}\end{pmatrix}. \tag{2.19}$$

The matrix $\mathcal{C}$ can also have three real eigenvalues, but with degenerate eigenspace. There are two inequivalent cases, leading to the follow two classes.

Class III: $\mathcal{C}$ is rank 2 and there is a twofold eigenvalue with a 1-dimension eigenspace. Such a $\mathcal{C}$ is isomorphic to

$$\mathcal{C}=\begin{pmatrix}\tilde{Q}+\alpha & \tilde{Q} & 0\\ -\tilde{Q} & -\tilde{Q}+\alpha & 0\\ 0 & 0 & -2\alpha\end{pmatrix}. \tag{2.20}$$

Note that we can fix $\tilde{Q}=\pm 1$ by an $O(1,1)$ boost. The corresponding matrix $\mathcal{M}$ is given by

$$\mathcal{M}=\begin{pmatrix}-e^{\alpha\rho}(1+\tilde{Q}\rho) & -e^{\alpha\rho}\tilde{Q}\rho & 0\\ -e^{\alpha\rho}\tilde{Q}\rho & e^{\alpha\rho}(1-\tilde{Q}\rho) & 0\\ 0 & 0 & e^{-2\alpha\rho}\end{pmatrix}. \tag{2.21}$$

Class IV: $\mathcal{C}$ is rank 2 and all of its eigenvalues are zero. Such a $\mathcal{C}$ is isomorphic to

$$\mathcal{C}=\begin{pmatrix}\cos\beta & \cos\beta & -\frac{1}{2}\sin\beta\\ -\cos\beta & -\cos\beta & \frac{1}{2}\sin\beta\\ \frac{1}{2}\sin\beta & \frac{1}{2}\sin\beta & 0\end{pmatrix}. \tag{2.22}$$

The matrix $\mathcal{M}$ is given by

$$\mathcal{M}=\begin{pmatrix}-(1+\rho\cos\beta-\frac{1}{8}\rho^2\sin^2\beta) & -(\rho\cos\beta-\frac{1}{8}\rho^2\sin^2\beta) & \frac{1}{2}\rho\sin\beta\\ -(\rho\cos\beta-\frac{1}{8}\rho^2\sin^2\beta) & (1-\rho\cos\beta+\frac{1}{8}\rho^2\sin^2\beta) & \frac{1}{2}\rho\sin\beta\\ \frac{1}{2}\rho\sin\beta & \frac{1}{2}\rho\sin\beta & 1\end{pmatrix}. \tag{2.23}$$

The classification of $\tilde{C}$ and $\tilde{\mathcal{M}}$, on the other hand, is somewhat different than those discussed in [7]. We may still use the rigid rescaling and the rigid gauge symmetry to make

$$e^{\Phi}(\rho_0)=1\,,\qquad \chi(\rho_0)\equiv A_{123}(\rho_0)=0 \tag{2.24}$$

at any but a specific point ρ_0. We shall choose it to be the asymptotic infinity, which is located at $\rho=0$. It follows that

$$\tilde{\mathcal{M}}(0)=\mathrm{diag}\{-1,1\}\,. \tag{2.25}$$

This ensures that the solutions are asymptotic flat. We are now left with an $SO(1,1)$ residual symmetry. However, unlike the previous $SL(3,\mathbb{R})$ case, in which the $SL(3,\mathbb{R})$ is

part of the general coordinate transformations in $D = 11$, we have no reason to use this residual symmetry further to put $\tilde{\mathcal{C}}$ into the simpler canonical form obtained in [7]. Instead, we should regard this $SO(1,1)$ as a nontrivial solution-generating symmetry group and write down all of the solutions. In practice, it is convenient to use the canonical forms obtained in [7] and then recover the full sets of inequivalent solutions by performing the $SO(1,1)$ transformations. This leads to the following three classes of solutions.

Class i: $\tilde{\mathcal{C}}$ has a pair of complex eigenvalues. Its canonical form is

$$\tilde{\mathcal{C}} = \begin{pmatrix} 0 & -\gamma \\ \gamma & 0 \end{pmatrix} . \tag{2.26}$$

The corresponding $\tilde{\mathcal{M}}$ is

$$\tilde{\mathcal{M}} = \begin{pmatrix} -\cos\gamma\rho & \sin\gamma\rho \\ \sin\gamma\rho & \cos\gamma\rho \end{pmatrix} . \tag{2.27}$$

To restore the full $SO(1,1)$ multiplets, we perform a further $SO(1,1)$ transformation with

$$\tilde{\Lambda} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} , \tag{2.28}$$

where θ is an arbitrary constant. The new $\tilde{\mathcal{M}}$ is then given by

$$\tilde{\mathcal{M}} \to \tilde{\Lambda}^T \tilde{\mathcal{M}} \tilde{\Lambda} = \begin{pmatrix} -\cos\gamma\rho + \sin\gamma\rho\ \sinh 2\theta & \sin\gamma\rho\ \cosh 2\theta \\ \sin\gamma\rho\ \cosh 2\theta & \cos\gamma\rho + \sin\gamma\rho\ \sinh 2\theta \end{pmatrix} . \tag{2.29}$$

Thus we have

$$\begin{aligned} e^{-\Phi} &= \cos\gamma\rho + \sin\gamma\rho\ \sinh 2\theta \\ \chi &= \frac{\sin\gamma\rho\ \cosh 2\theta}{\cos\gamma\rho + \sin\gamma\rho\ \sinh 2\theta} \end{aligned} \tag{2.30}$$

Class ii: $\tilde{\mathcal{C}}$ has two real eigenvalues, with one timelike and one spacelike eigenvectors. Its canonical form is

$$\tilde{\mathcal{C}} = \begin{pmatrix} \gamma & 0 \\ 0 & -\gamma \end{pmatrix} , \tag{2.31}$$

and hence

$$\tilde{\mathcal{M}} = \begin{pmatrix} -e^{\gamma\rho} & 0 \\ 0 & e^{-\gamma\rho} \end{pmatrix} . \tag{2.32}$$

A further $SO(1,1)$ transformation leads to

$$\tilde{\mathcal{M}} \to \tilde{\Lambda}^T \tilde{\mathcal{M}} \tilde{\Lambda} = \begin{pmatrix} -(\cosh\gamma\rho + \sinh\gamma\rho\ \cosh 2\theta) & -\sinh\gamma\rho\ \sinh 2\theta \\ -\sinh\gamma\rho\ \sinh 2\theta & \cosh\gamma\rho - \sinh\gamma\rho\ \sinh 2\theta \end{pmatrix} . \tag{2.33}$$

Thus, we have

$$
\begin{aligned}
e^{-\Phi} &= \cosh\gamma\rho - \sinh\gamma\rho\ \cosh 2\theta \\
\chi &= -\frac{\sinh\gamma\rho\ \sinh 2\theta}{\cosh\gamma\rho - \sinh\gamma\rho\ \cosh 2\theta}
\end{aligned}
\tag{2.34}
$$

Class iii: $\tilde{\mathcal{C}}$ is rank 1 and all of its eigenvalues are zero. Its canonical form is

$$
\tilde{\mathcal{C}} = \begin{pmatrix} Q & Q \\ -Q & -Q \end{pmatrix} , \tag{2.35}
$$

which yields

$$
\tilde{\mathcal{M}} = \begin{pmatrix} -(1+Q\rho) & -Q\rho \\ -Q\rho & 1-Q\rho \end{pmatrix} . \tag{2.36}
$$

The further $SO(1,1)$ transformation leads to

$$
\tilde{\mathcal{M}} \to \tilde{\Lambda}^T \tilde{\mathcal{M}} \tilde{\Lambda} = \begin{pmatrix} -(1+e^{2\theta}Q\rho) & -e^{2\theta}Q\rho \\ -e^{2\theta}Q\rho & 1-e^{2\theta}Q\rho \end{pmatrix} . \tag{2.37}
$$

Thus we see that θ can be absorbed into the redefinition of Q. Thus, in general we have

$$
e^{-\Phi} = 1 - Q\rho\,, \qquad \chi = -\frac{Q\rho}{1-Q\rho} . \tag{2.38}
$$

We have thus obtained the solutions for both $\mathcal{M}$ and $\tilde{\mathcal{M}}$. The combination of the two gives rise to a total of 12 classes of solutions. It is straightforward to lift these solutions back to $D=11$ using the reduction ansatz discussed in section (2.1). These 12 classes of solutions comprise the most general spherically symmetric M2-branes, with the isometry group of $\mathbb{R}^{1,2} \times SO(8)$.

2.3 Properties of the solutions

In the previous subsection, we have shown that there exist twelve different classes of spherically symmetric M2-branes. We shall now discuss their properties. To do this, we first make a coordinate transformation so that the metric (2.7) is manifestly conformal flat, namely,

$$
ds_8^2 = g(\tilde{r})(d\tilde{r}^2 + \tilde{r}^2 d\Omega_7^2) = (1 + \frac{a}{4\tilde{r}^{12}})^{\frac{1}{3}}(d\tilde{r}^2 + \tilde{r}^2 d\Omega_7^2)\,, \tag{2.39}
$$

where

$$
\tilde{r}^6 = \tfrac{1}{2}(r^6 + \sqrt{r^6 - a}\,) . \tag{2.40}
$$

Thus we have

$$
r^6 = \frac{4\tilde{r}^{12} + a}{4\tilde{r}^6}\,, \qquad f = \frac{(1 - \frac{a}{4\tilde{r}^{12}})^2}{(1 + \frac{a}{4\tilde{r}^{12}})^2}\,, \qquad \cos(6\sqrt{a}\rho) = \frac{1 - \frac{a}{4\tilde{r}^{12}}}{1 + \frac{a}{4\tilde{r}^{12}}} . \tag{2.41}
$$

For $a \le 0$, there will be a naked curvature singularity at $\tilde{r}^{12} = -a/4$ in general. Note that the point where $e^{-\Phi} = 0$ is also singular since the volume of the world volume becomes infinite while that of the transverse space shrinks to zero. To see this, let us suppose that $e^{-\Phi} = 0$ for a certain finite $\tilde{r} = \tilde{r}_0 > 0$, and we find that both $\partial_\rho e^{-\Phi}$ and $\partial_{\tilde{r}}\rho$ are finite at this point in all classes of the solutions. It follows that the metric must be singular at this point, since

$$e^{-\frac{\Phi}{3}} ds_8^2 \sim (\tilde{r} - \tilde{r}_0)^{\frac{1}{3}} (d\tilde{r}^2 + \tilde{r}_0^2 d\Omega_7^2)\,. \tag{2.42}$$

When such a singularity is avoided by an appropriate choice of the parameters with a proper value of γ, we would have regular solutions if $\tilde{r}^{12} = -a/4$ is a degenerate surface and thus it is just a coordinate singularity. Such a solution is either a black hole or a Kaluza-Klein bubble.

For $a > 0$, $\tilde{r}^{12} = a/4$ could be just a coordinate singularity and the metric can be regular if there are no further singular terms from e^{Φ}, which can be avoided if the point $\coth \gamma\rho = -\cosh 2\theta$ (or $\cot \gamma\rho = \sinh 2\theta$) is out of the range of our parameter except the asymptotic point $\tilde{r} = 0$. In this case, the range of $\tilde{r}$ can be extended to $(0, \infty)$. Note that (2.39) and (2.41) are invariant under

$$\tilde{r}^6 \to \frac{a}{4\tilde{r}^6}\,, \tag{2.43}$$

and thus (2.39) describes a wormhole connecting two asymptotic regions W_0 and W_∞ at $\tilde{r} = 0$ and ∞ respectively. From now on, we shall denote $\tilde{r}$ by r for simplicity.

Class I.i

This class of solutions is the one that combines Class I for $\mathcal{M}$ and Class i for $\tilde{\mathcal{M}}$. Lifting the solutions back to $D = 11$, we have

$$\begin{aligned} ds^2 &= e^{\frac{2}{3}\Phi}\left(e^{\alpha\rho}\cos\beta\rho(dz_1^2 - dt^2) + 2e^{\alpha\rho}\sin\beta\rho\, dt dz_1 + e^{-2\alpha\rho} dz_2^2\right) \\ &\quad + e^{-\frac{1}{3}\Phi}(1 + \tfrac{a}{4r^{12}})^{\frac{1}{3}}(dr^2 + r^2 d\Omega_7^2)\,, \\ A_{(3)} &= \frac{\sin\gamma\rho\cosh 2\theta}{\cos\gamma\rho + \sin\gamma\rho\sinh 2\theta} dt\wedge dz^1 \wedge dz^2\,, \end{aligned} \tag{2.44}$$

where

$$\begin{aligned} & e^{-\Phi} = \cos\gamma\rho + \sin\gamma\rho\sinh 2\theta\,, \qquad \cos(6\sqrt{a}\rho) = \frac{1 - \frac{a}{4r^{12}}}{1 + \frac{a}{4r^{12}}}\,, \\ & \beta^2 + \gamma^2 - 3\alpha^2 = 84a\,. \end{aligned} \tag{2.45}$$

If $a \le 0$, then $(-a/4)^{\frac{1}{12}} < r < \infty$ corresponds to $-\infty < \rho < 0$. There is always a naked curvature singularity at $\cot\gamma\rho = -\sinh 2\theta$. If $a > 0$, the range for r is $0 < r < \infty$,

corresponding to $-\pi < 6\sqrt{a}\rho < 0$. The point $r = (a/4)^{\frac{1}{12}}$ corresponds to $6\sqrt{a}\rho = \pi/2$. To avoid the possible curvature singularity at $\cot\gamma\rho = -\sinh 2\theta$, we must have

$$\begin{cases} -\frac{\pi\gamma}{6\sqrt{a}} \geq -\mathrm{arccot}(\sinh 2\theta) > -\pi \qquad \text{for} \quad \gamma > 0\ , \\ -\frac{\pi\gamma}{6\sqrt{a}} \leq \mathrm{arccot}(-\sinh 2\theta) < \pi \qquad \text{for} \quad \gamma < 0\ . \end{cases} \tag{2.46}$$

The solutions then describe smooth M2-brane wormholes that connect two asymptotic regions. One asymptotic region lies at $r \to \infty$, for which we have

$$\rho \to -\frac{1}{6r^6}\ , \qquad e^{-\Phi} \to 1 - \frac{\gamma \sinh 2\theta}{6r^6}\ . \tag{2.47}$$

It follows that

$$\begin{aligned} ds^2 \to\ & \left(1 + \left(\tfrac{2}{3}\gamma\sinh 2\theta - \alpha\right)\frac{1}{6r^6}\right)(-dt^2 + dz_1^2) - \frac{2\beta}{6r^6}dtdz_1 \\ & + \left(1 + \left(\tfrac{2}{3}\gamma\sinh 2\theta + 2\alpha\right)\frac{1}{6r^6}\right)dz_2^2 + \left(1 - \frac{\gamma\sinh 2\theta}{18r^6}\right)(dr^2 + r^2 d\Omega_7^2)\,, \\ A_{(3)} \to\ & -\frac{\gamma\cosh 2\theta}{6r^6} dt \wedge dz^1 \wedge dz^2\ . \end{aligned} \tag{2.48}$$

We shall call this asymptotic region W_∞, which is the eleven-dimensional Mankowski spacetime. The ADM mass can be calculated from the subleading terms of this asymptotic region, given by

$$M = \int_{r\to\infty} d^7\Sigma^m(\partial_n h_{mn} - \partial_m h_{ii}) = \Omega_7(\alpha - \gamma\sinh 2\theta)\,, \tag{2.49}$$

where

$$\Omega_{p-1} = \frac{2\pi^{\frac{p}{2}}}{\Gamma(\frac{p}{2})} \tag{2.50}$$

is the volume of the unit sphere S^p. The momentum along the z_1 direction is

$$P_1 = \beta\Omega_7\,. \tag{2.51}$$

The M2-brane charge is

$$Q_{M2} = \int_{r\to\infty} *F^{(4)} = \int_{r\to\infty} d^7\Sigma^m F_{m012} = \gamma\cosh(2\theta)\,\Omega_7\,. \tag{2.52}$$

The other asymptotic region is at $r = 0$, which we shall denote as W_0. When the inequality of (2.46) is strictly held, W_0 is an asymptotically flat region. To see this, let us define $R^6 = a/4r^6$. For $r \to 0$, we have

$$\begin{aligned} \rho &\to \frac{1}{6R^6} - \frac{\pi}{6\sqrt{a}}\ , \\ e^{-\Phi} &\to \left[\cos\left(\frac{\gamma\pi}{6\sqrt{a}}\right) - \sin\left(\frac{\gamma\pi}{6\sqrt{a}}\right)\sinh 2\theta\right]\left[1 + \frac{\sin\left(\frac{\gamma\pi}{6\sqrt{a}}\right) + \cos\left(\frac{\gamma\pi}{6\sqrt{a}}\right)\sinh 2\theta}{\cos\left(\frac{\gamma\pi}{6\sqrt{a}}\right) - \sin\left(\frac{\gamma\pi}{6\sqrt{a}}\right)\sinh 2\theta}\frac{\gamma}{6R^6}\right] \end{aligned}$$

$$= A(1+\frac{\gamma B}{6R^6})\,, \tag{2.53}$$

where

$$A = \cos(\frac{\gamma\pi}{6\sqrt{a}}) - \sin(\frac{\gamma\pi}{6\sqrt{a}})\sinh 2\theta\ , \qquad B = \frac{\sin(\frac{\gamma\pi}{6\sqrt{a}}) + \cos(\frac{\gamma\pi}{6\sqrt{a}})\sinh 2\theta}{\cos(\frac{\gamma\pi}{6\sqrt{a}}) - \sin(\frac{\gamma\pi}{6\sqrt{a}})\sinh 2\theta}\,. \tag{2.54}$$

Making the follow coordinate redefinition

$$\begin{aligned}
\tilde{z}_1 &= A^{-\frac{1}{3}} e^{-\frac{\alpha\pi}{12\sqrt{a}}} \left[z_1 \cos\left(\frac{\beta\pi}{12\sqrt{a}}\right) - t\sin\left(\frac{\beta\pi}{12\sqrt{a}}\right)\right]\,,\\
\tilde{t} &= A^{-\frac{1}{3}} e^{-\frac{\alpha\pi}{12\sqrt{a}}} \left[z_1 \sin\left(\frac{\beta\pi}{12\sqrt{a}}\right) + t\cos\left(\frac{\beta\pi}{12\sqrt{a}}\right)\right]\,,\\
\tilde{z}_2 &= A^{-\frac{1}{3}} e^{\frac{\alpha\pi}{6\sqrt{a}}} z_2\,,
\end{aligned} \tag{2.55}$$

we have

$$\begin{aligned}
ds^2 &\to \left[1 - (\tfrac{2}{3}\gamma B - \alpha)\frac{1}{6R^6}\right](-d\tilde{t}^2 + d\tilde{z}_1^2) + \frac{2\beta}{6R^6} d\tilde{t} d\tilde{z}_1 + \left[1 - (\tfrac{2}{3}\gamma B + 2\alpha)\frac{1}{6R^6}\right] d\tilde{z}_2^2\\
&\quad + (1+\frac{\gamma B}{18R^6})A^{\frac{1}{3}}(dR^2 + R^2 d\Omega_7^2)\,,\\
A_{(3)} &\to -A^{-1}\cosh 2\theta \sin\left(\frac{\gamma\pi}{\sqrt{a}}\right)\left[1 - \gamma\left(\cot\left(\frac{\gamma\pi}{6\sqrt{a}}\right) + B\right)\frac{1}{6R^6}\right] dt\wedge dz^1 \wedge dz^2\\
&= -\cosh 2\theta \sin\left(\frac{\gamma\pi}{6\sqrt{a}}\right)\left[1 - \frac{\gamma}{6A\,R^6 \sin(\frac{\gamma\pi}{6\sqrt{a}})}\right] d\tilde{t}\wedge d\tilde{z}^1\wedge d\tilde{z}^2\,.
\end{aligned} \tag{2.56}$$

The ADM mass, momentum, and M2 charges, measured in W_0, are given by, respectively,

$$M = (\gamma B - \alpha) A\,\Omega_7, \qquad P_1 = -\beta A\,\Omega_7\,, \qquad Q_{M2} = -\gamma\cosh(2\theta)\,\Omega_7\,. \tag{2.57}$$

If, instead, the equality in (2.46) is held, namely,

$$\begin{cases} -\frac{\gamma\pi}{6\sqrt{a}} = -\mathrm{arccot}(\sinh 2\theta) > -\pi & \text{for} \quad \gamma > 0\ ,\\ -\frac{\gamma\pi}{6\sqrt{a}} = \mathrm{arccot}(-\sinh 2\theta) < \pi & \text{for} \quad \gamma < 0\ , \end{cases} \tag{2.58}$$

the asymptotic region W_0 becomes $\mathrm{AdS}_4 \times S^7$. This can be seen easily since we have

$$e^{-\Phi} \to \gamma \sin\left(\frac{\gamma\pi}{6\sqrt{a}}\right)\cosh^2 2\theta \frac{1}{6R^6}\,. \tag{2.59}$$

Class I.ii

The solution is

$$ds^2 = e^{\frac{2}{3}\Phi}(e^{\alpha\rho}\cos\beta\rho(dz_1^2 - dt^2) + 2e^{\alpha\rho}\sin\beta\rho\, dt dz_1 + e^{-2\alpha\rho} dz_2^2)$$

$$
\begin{aligned}
&\quad +e^{-\frac{1}{3}\Phi}(1+\frac{a}{4r^{12}})^{\frac{1}{3}}(dr^2+r^2d\Omega_7^2)\,,\\
A_{(3)} &= -\frac{\sinh\gamma\rho\ \sinh 2\theta}{\cosh\gamma\rho-\sinh\gamma\rho\ \cosh 2\theta}dt\wedge dz^1\wedge dz^2\,,
\end{aligned}
\tag{2.60}
$$

where

$$
\begin{aligned}
&e^{-\Phi}=\cosh\gamma\rho-\sinh\gamma\rho\ \cosh 2\theta\,, \qquad \cos(6\sqrt{a}\rho)=\frac{1-\frac{a}{4r^{12}}}{1+\frac{a}{4r^{12}}}\,,\\
&\beta^2-\gamma^2-3\alpha^2=84a\,.
\end{aligned}
\tag{2.61}
$$

In the case of $a\le 0$, the coordinate r lies in the range of $(-a/4)^{\frac{1}{12}}<r<\infty$, corresponding to $-\infty<\rho<0$. For $\gamma<0$, there is curvature singularity at $\coth\gamma\rho=\cosh 2\theta$, and the range of ρ lies in $\gamma^{-1}\mathrm{arccoth}(\cosh 2\theta)<\rho<0$. For $\gamma\ge 0$, the range of ρ is $-\infty<\rho<0$, and $r=(-a/4)^{\frac{1}{12}}$ is a naked singularity in general, except for the case of the M2-brane bubble, arising at

$$
\alpha=-\tfrac{2}{3}\gamma\,,\qquad \frac{\gamma}{\sqrt{-a}}=6\,,\qquad \beta=0\,. \tag{2.62}
$$

The solution is a double Wick rotation of a black M2-brane [21], analogous to the Kaluza-Klein bubble which is a double Wick rotation of a black hole.

In the case of $a>0$, we have $0<r<\infty$, corresponding to $-\pi<6\sqrt{a}\rho<0$. The point $r=(a/4)^{\frac{1}{12}}$ corresponds to $6\sqrt{a}\rho=\pi/2$. To avoid the possible curvature singularity at $\coth\gamma\rho=\cosh 2\theta$, we must have

$$
-\frac{\gamma\pi}{6\sqrt{a}}\le \mathrm{arccoth}(\cosh 2\theta)\,. \tag{2.63}
$$

The solutions describe wormholes that connect two asymptotic regions W_∞ and W_0 at $r=\infty$ and $r=0$, respectively.

As $r\to\infty$, we have

$$
\rho\to-\frac{1}{6r^6}\,,\qquad e^{-\Phi}\to 1+\frac{\gamma\cosh 2\theta}{6r^6}\,, \tag{2.64}
$$

and then

$$
\begin{aligned}
ds^2 &\to \Big[1-\big(\tfrac{2}{3}\gamma\cosh 2\theta+\alpha\big)\frac{1}{6r^6}\Big](-dt^2+dz_1^2)-\frac{2\beta}{6r^6}dtdz_1\\
&\quad+\Big[1-\big(\tfrac{2}{3}\gamma\cosh 2\theta-2\alpha\big)\frac{1}{6r^6}\Big]dz_2^2+(1+\frac{\gamma\cosh 2\theta}{18r^6})(dr^2+r^2d\Omega_7^2)\,,\\
A_{(3)} &\to \frac{\gamma\sinh 2\theta}{6r^6}dt\wedge dz^1\wedge dz^2\,.
\end{aligned}
\tag{2.65}
$$

Thus the asymptotic region W_∞ is flat. The ADM mass, momentum, and M2 charges are given by, respectively,

$$
M=(\alpha+\gamma\cosh 2\theta)\Omega_7\,,\qquad P_1=\beta\Omega_7\,,\qquad Q_{M2}=-\gamma\sinh(2\theta)\Omega_7\,. \tag{2.66}
$$

We now look at the asymptotic region W_0, which is also flat if the inequality (2.63) is strictly held. Let us define $R^6 = a/4r^6$, and we have

$$
\begin{aligned}
\rho &\to \frac{1}{6R^6} - \frac{\pi}{6\sqrt{a}} \,, \\
e^{-\Phi} &\to \Big[\cosh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big) + \sinh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big)\cosh 2\theta\Big]\left[1 - \frac{\sinh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big) + \cosh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big)\cosh 2\theta}{\cosh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big) + \sinh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big)\cosh 2\theta}\frac{\gamma}{6R^6}\right] \\
&= A\big(1 + \frac{\gamma B}{6R^6}\big) \,,
\end{aligned}
\tag{2.67}
$$

where

$$
A = \cosh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big) + \sinh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big)\cosh 2\theta \,, \qquad B = -\frac{\sinh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big) + \cosh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big)\cosh 2\theta}{\cosh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big) + \sinh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big)\sinh 2\theta} \,. \tag{2.68}
$$

Introducing new asymptotic coordinates

$$
\begin{aligned}
\tilde{z}_1 &= A^{-\frac{1}{3}} e^{-\frac{\alpha\pi}{12\sqrt{a}}}\Big[z_1\cos\big(\tfrac{\beta\pi}{12\sqrt{a}}\big) - t\sin\big(\tfrac{\beta\pi}{12\sqrt{a}}\big)\Big] \,, \\
\tilde{t} &= A^{-\frac{1}{3}} e^{-\frac{\alpha\pi}{12\sqrt{a}}}\Big[z_1\sin\big(\tfrac{\beta\pi}{12\sqrt{a}}\big) + t\cos\big(\tfrac{\beta\pi}{12\sqrt{a}}\big)\Big] \,, \\
\tilde{z}_2 &= A^{-\frac{1}{3}} e^{\frac{\alpha\pi}{6\sqrt{a}}} z_2 \,,
\end{aligned}
\tag{2.69}
$$

we obtain

$$
\begin{aligned}
ds^2 &\to \Big[1 - \big(\tfrac{2}{3}\gamma B - \alpha\big)\frac{1}{6R^6}\Big](-d\tilde{t}^2 + d\tilde{z}_1^2) + \frac{\beta}{3R^6} d\tilde{t} d\tilde{z}_1 + \Big[1 - \big(\tfrac{2}{3}\gamma B + 2\alpha\big)\frac{1}{6R^6}\Big] d\tilde{z}_2^2 \\
&\quad + \big(1 + \frac{\gamma B}{18R^6}\big) A^{\frac{1}{3}} (dR^2 + R^2 d\Omega_7^2) \,, \\
A_{(3)} &\to A^{-1}\sinh 2\theta \sinh\left(\frac{\gamma\pi}{6\sqrt{a}}\right)\left[1 - \left(\coth\left(\frac{\gamma\pi}{6\sqrt{a}}\right) + B\right)\frac{\gamma}{6R^6}\right] dt \wedge dz^1 \wedge dz^2 \\
&= \sinh 2\theta \sinh\left(\frac{\gamma\pi}{6\sqrt{a}}\right)\left[1 - \frac{\gamma}{6AR^6\sinh\big(\tfrac{\gamma\pi}{6\sqrt{a}}\big)}\right] d\tilde{t} \wedge d\tilde{z}^1 \wedge d\tilde{z}^2 \,.
\end{aligned}
\tag{2.70}
$$

Indeed, the metric is flat when $R \to \infty$. The ADM mass, momentum and M2 charges are given by, respectively,

$$
M = (\gamma B - \alpha) A \Omega_7 \,, \qquad P_1 = -\beta A \Omega_7 \,, \qquad Q_{M2} = \gamma \sinh(2\theta)\,\Omega_7 \,. \tag{2.71}
$$

If, on the other hand, the equality in (2.63) is held, then W_0 is asymptotically $\mathrm{AdS}_4 \times S^7$. This is because

$$
e^{-\Phi} \to \sinh\left(\frac{\gamma\pi}{6\sqrt{a}}\right)\sinh^2 2\theta \frac{\gamma}{6R^6} \,. \tag{2.72}
$$

Class I.iii

The solution is

$$
\begin{aligned}
ds^2 &= e^{\frac{2}{3}\Phi}(e^{\alpha\rho}\cos\beta\rho(dz_1^2 - dt^2) + 2e^{\alpha\rho}\sin\beta\rho dt dz_1 + e^{-2\alpha\rho}dz_2^2) \\
&\quad + e^{-\frac{1}{3}\Phi}(1+\frac{a}{4r^{12}})^{\frac{1}{3}}(dr^2 + r^2 d\Omega_7^2)\,, \\
A_{(3)} &= -\frac{Q\rho}{1-Q\rho} dt\wedge dz^1 \wedge dz^2
\end{aligned}
\tag{2.73}
$$

where

$$
e^{-\Phi} = 1 - Q\rho\,, \qquad \cos(6\sqrt{a}\rho) = \frac{1-\frac{a}{4r^{12}}}{1+\frac{a}{4r^{12}}}\,, \qquad \beta^2 - 3\alpha^2 = 84a\,. \tag{2.74}
$$

In the case of $a \le 0$, we have $(-a/4)^{\frac{1}{12}} < r < \infty$, and correspondingly, the range of ρ is $-\infty < \rho < 0$. For $Q < 0$, there is a curvature singularity at $Q\rho = 1$, and the range of ρ is $Q^{-1} < \rho < 0$. For $Q \ge 0$, the range of ρ is $-\infty < \rho < 0$, and $r = (-a/4)^{\frac{1}{12}}$ is singular, except for the extremal case $\alpha = \beta = 0$.

In the case of $a > 0$, we have $0 < r < \infty$, and hence $-\pi < 6\sqrt{a}\rho < 0$. The point of $r = (a/4)^{\frac{1}{12}}$ corresponds to $6\sqrt{a}\rho = \pi/2$. To avoid the possible curvature singularity associated with singular e^{Φ}, we need

$$
-\frac{Q\pi}{6\sqrt{a}} \le 1\,. \tag{2.75}
$$

The asymptotic region W_∞ at $r \to \infty$ is flat. To see this, we note that

$$
\rho \to -\frac{1}{6r^6}\,, \qquad e^{-\Phi} \to 1 + \frac{Q}{6r^6}\,. \tag{2.76}
$$

It follows that

$$
\begin{aligned}
ds^2 &\to \left(1 - \left(\tfrac{2}{3}Q + \alpha\right)\frac{1}{6r^6}\right)(-dt^2 + dz_1^2) - \frac{2\beta}{6r^6}dt dz_1 + \left(1 - (\tfrac{2}{3}Q - 2\alpha)\frac{1}{6r^6}\right)dz_2^2 \\
&\quad + (1 + \frac{Q}{18r^6})(dr^2 + r^2 d\Omega_7^2)\,, \\
A_{(3)} &\to \frac{Q}{6r^6} dt\wedge dz^1\wedge dz^2\,.
\end{aligned}
\tag{2.77}
$$

The ADM mass, momentum, and M2 charges are given by, respectively

$$
M = (\alpha + Q)\Omega_7\,, \qquad P_1 = \beta\Omega_7\,, \qquad Q_{M2} = -Q\Omega_7\,. \tag{2.78}
$$

The asymptotic region W_0 at $r = 0$ is also flat, provided that the inequality of (2.75) is strictly held. Let us define $R^6 = a/4r^6$, and then

$$
\begin{aligned}
\rho &\to \frac{1}{6R^6} - \frac{\pi}{6\sqrt{a}}\,, \\
e^{-\Phi} &\to 1 + \frac{Q\pi}{6\sqrt{a}} - \frac{Q}{6R^6} = A(1 - \frac{Q}{6AR^6})\,,
\end{aligned}
\tag{2.79}
$$

where

$$A = 1 + \frac{Q\pi}{6\sqrt{a}}\,. \tag{2.80}$$

Further introducing new asymptotic coordinates

$$\begin{aligned}
\tilde{z}_1 &= A^{-\frac{1}{3}} e^{-\frac{\alpha\pi}{12\sqrt{a}}} \Big(z_1 \cos\big(\tfrac{\beta\pi}{12\sqrt{a}}\big) - t \sin\big(\tfrac{\beta\pi}{12\sqrt{a}}\big)\Big) \,, \\
\tilde{t} &= A^{-\frac{1}{3}} e^{-\frac{\alpha\pi}{12\sqrt{a}}} \Big(z_1 \sin\big(\tfrac{\beta\pi}{12\sqrt{a}}\big) + t \cos\big(\tfrac{\beta\pi}{12\sqrt{a}}\big)\Big) \,, \\
\tilde{z}_2 &= A^{-\frac{1}{3}} e^{\frac{\alpha\pi}{6\sqrt{a}}} z_2 \,,
\end{aligned} \tag{2.81}$$

we have

$$\begin{aligned}
ds^2 &\to \Big(1 + (\tfrac{2Q}{3A} + \alpha)\frac{1}{6R^6}\Big)(-d\tilde{t}^2 + d\tilde{z}_1^2) + \frac{2\beta}{6R^6} d\tilde{t} d\tilde{z}_1 + \Big(1 + (\tfrac{2Q}{3A} - 2\alpha)\frac{1}{6R^6}\Big) d\tilde{z}_2^2 \\
&\quad + (1 - \frac{Q}{18AR^6}) A^{\frac{1}{3}} (dR^2 + R^2 d\Omega_7^2) \,, \\
A_{(3)} &\to \frac{Q\pi}{6\sqrt{a}} \Big(1 - \frac{\sqrt{a}}{\pi A R^6}\Big) d\tilde{t} \wedge d\tilde{z}^1 \wedge d\tilde{z}^2 \,.
\end{aligned} \tag{2.82}$$

The ADM mass, momentum, and M2 charges are given by, respectively,

$$M = -(Q + \alpha A)\Omega_7 \ , \qquad P_1 = -\beta A \Omega_7 \ , \qquad Q_{M2} = Q\Omega_7 \,. \tag{2.83}$$

If on the other hand, the equality in (2.75) is held, we have

$$e^{-\Phi} \to -\frac{Q}{6R^6}\,. \tag{2.84}$$

It follows that the asymptotic region W_0 is $\mathrm{AdS}_4 \times S^7$. This solution was obtained and discussed in detail in [22].

Thus we see that all class I.i, class I.ii and class I.iii solutions contain M2-brane wormholes that smoothly connect two asymptotic regions. One asymptotic region is always flat, while the other can be either flat or $\mathrm{AdS}_4 \times S^7$.

Class II.i

The solution is

$$\begin{aligned}
ds^2 &= e^{\frac{2}{3}\Phi} \Big(-e^{\alpha_1 \rho} dt^2 + e^{\alpha_2 \rho} dz_1^2 + e^{-(\alpha_1 + \alpha_2)\rho} dz_2^2 \Big) + e^{-\frac{1}{3}\Phi} (1 + \frac{a}{4r^{12}})^{\frac{1}{3}} (dr^2 + r^2 d\Omega_7^2) \,, \\
A_{(3)} &= \frac{\sin\gamma\rho \cosh 2\theta}{\cos\gamma\rho + \sin\gamma\rho \sinh 2\theta} dt \wedge dz^1 \wedge dz^2 \,,
\end{aligned} \tag{2.85}$$

where

$$e^{-\Phi} = \cos\gamma\rho + \sin\gamma\rho \sinh 2\theta \ , \qquad \cos(6\sqrt{a}\rho) = \frac{1 - \frac{a}{4r^{12}}}{1 + \frac{a}{4r^{12}}} \ ,$$

$$\gamma^2 - \alpha_1^2 - \alpha_2^2 - \alpha_1\alpha_2 = 84a \ . \tag{2.86}$$

If $a \le 0$, $(-a/4)^{\frac{1}{12}} < r < \infty$ corresponds to $-\infty < \rho < 0$. There is always a naked curvature singularity at $\cot\gamma\rho = -\sinh 2\theta$. If $a > 0$, $0 < r < \infty$ corresponds to $-\pi < 6\sqrt{a}\rho < 0$ and $r = (a/4)^{\frac{1}{12}}$ corresponds to $6\sqrt{a}\rho = \pi/2$. To avoid the possible curvature singularity at $\cot\gamma\rho = -\sinh 2\theta$, we must have

$$\begin{cases} -\frac{\gamma\pi}{6\sqrt{a}} \ge -\mathrm{arccot}(\sinh 2\theta) > -\pi & \text{for} \quad \gamma > 0 \ , \\ -\frac{\gamma\pi}{6\sqrt{a}} \le \mathrm{arccot}(-\sinh 2\theta) < \pi & \text{for} \quad \gamma < 0 \ . \end{cases} \tag{2.87}$$

However, from the constraint, we find that $|\frac{\gamma}{\sqrt{a}}| > 2\sqrt{21} > 6$. Thus the naked curvature singularity at $\cot\gamma\rho = -\sinh 2\theta$ is unavoidable, and there is no regular solution in this class. The special case with $\alpha_1 = \alpha_2 = 0$ was obtained and discussed in detail in [20].

Class II.ii

The solution is

$$\begin{aligned} ds^2 &= e^{\frac{2}{3}\Phi}\left(-e^{\alpha_1\rho}dt^2 + e^{\alpha_2\rho}dz_1^2 + e^{-(\alpha_1+\alpha_2)\rho}dz_2^2\right) + e^{-\frac{1}{3}\Phi}(1 + \frac{a}{4r^{12}})^{\frac{1}{3}}(dr^2 + r^2 d\Omega_7^2) \, , \\ A_{(3)} &= -\frac{\sinh\gamma\rho \ \sinh 2\theta}{\cosh\gamma\rho - \sinh\gamma\rho \ \cosh 2\theta} dt \wedge dz^1 \wedge dz^2 \, , \end{aligned} \tag{2.88}$$

where

$$\begin{aligned} & e^{-\Phi} = \cosh\gamma\rho - \sinh\gamma\rho \ \cosh 2\theta \ , \qquad \cos(6\sqrt{a}\rho) = \frac{1 - \frac{a}{4r^{12}}}{1 + \frac{a}{4r^{12}}} \ , \\ & -\gamma^2 - \alpha_1^2 - \alpha_2^2 - \alpha_1\alpha_2 = 84a \ . \end{aligned} \tag{2.89}$$

In this class we always have $a \le 0$. Then the range $(-a/4)^{\frac{1}{12}} < r < \infty$ corresponds to $-\infty < \rho < 0$. In the case $\gamma < 0$, there is a curvature singularity at $\coth\gamma\rho = \cosh 2\theta$, and the range of ρ should be $\gamma^{-1}\mathrm{arccoth}(\cosh 2\theta) < \rho < 0$. In the case $\gamma \ge 0$, the range of ρ should be $-\infty < \rho < 0$, and $r = (-a/4)^{\frac{1}{12}}$ is singular in general, except in the cases when the black M2-brane (or its double Wick rotation counterpart M2-brane bubble (2.62) which is shared by classes I.ii and II.ii) arises, namely,

$$\alpha_1 = -2\alpha_2 \ , \quad \alpha_2 = -\tfrac{2}{3}\gamma \ , \qquad \frac{\gamma}{\sqrt{-a}} = 6 \, . \tag{2.90}$$

Another special case with $\alpha_1 = \alpha_2 = 0$ was obtained and discussed in detail in [19, 20].

Class II.iii

The solution is

$$ds^2 = e^{\frac{2}{3}\Phi}\left(-e^{\alpha_1\rho}dt^2 + e^{\alpha_2\rho}dz_1^2 + e^{-(\alpha_1+\alpha_2)\rho}dz_2^2\right) + e^{-\frac{1}{3}\Phi}(1+\frac{a}{4r^{12}})^{\frac{1}{3}}(dr^2 + r^2 d\Omega_7^2)\,,$$
$$A_{(3)} = -\frac{Q\rho}{1-Q\rho}dt\wedge dz^1\wedge dz^2\,, \tag{2.91}$$

where

$$e^{-\Phi} = 1 - Q\rho\ ,\qquad \cos(6\sqrt{a}\rho) = \frac{1-\frac{a}{4r^{12}}}{1+\frac{a}{4r^{12}}}\ ,$$
$$-\alpha_1^2 - \alpha_2^2 - \alpha_1\alpha_2 = 84a\ . \tag{2.92}$$

In this class we always have $a \le 0$. Then $(-a/4)^{\frac{1}{12}} < r < \infty$ corresponds to $-\infty < \rho < 0$. In the case $Q < 0$, there is a curvature singularity at $Q\rho = 1$, and the range of ρ should be $Q^{-1} < \rho < 0$. In the case $Q \ge 0$, the range of ρ should be $-\infty < \rho < 0$, and $r = (-a/4)^{\frac{1}{12}}$ is the singularity except for the extremal case $\alpha_1 = \alpha_2 = 0$, which corresponds to the BPS M2-brane [12].

Class III.i

The solution is

$$\begin{aligned} ds^2 &= e^{\frac{2}{3}\Phi}\left\{e^{\alpha\rho}\left[-(1+\tilde{Q}\rho)dt^2 - 2\tilde{Q}\rho dt dz_1 + (1-\tilde{Q}\rho)dz_1^2\right] + e^{-2\alpha\rho}dz_2^2\right\} \\ &\quad + e^{-\frac{1}{3}\Phi}(1+\frac{a}{4r^{12}})^{\frac{1}{3}}(dr^2 + r^2 d\Omega_7^2)\,, \\ A_{(3)} &= \frac{\sin\gamma\rho\cosh 2\theta}{\cos\gamma\rho + \sin\gamma\rho\sinh 2\theta}dt\wedge dz^1\wedge dz^2\,, \end{aligned} \tag{2.93}$$

where

$$e^{-\Phi} = \cos\gamma\rho + \sin\gamma\rho\sinh 2\theta\ ,\qquad \cos(6\sqrt{a}\rho) = \frac{1-\frac{a}{4r^{12}}}{1+\frac{a}{4r^{12}}}\ ,$$
$$\gamma^2 - 3\alpha^2 = 84a\ . \tag{2.94}$$

If $a \le 0$, the coordinate range $(-a/4)^{\frac{1}{12}} < r < \infty$ corresponds to $-\infty < \rho < 0$. There should always be a curvature singularity at $\cot\gamma\rho = -\sinh 2\theta$ where ρ is finite. If $a > 0$, $0 < r < \infty$ corresponds to $-\pi < 6\sqrt{a}\rho < 0$, and $r = (a/4)^{\frac{1}{12}}$ corresponds to $6\sqrt{a}\rho = \pi/2$. To avoid the possible curvature singularity at $\cot\gamma\rho = -\sinh 2\theta$, we need

$$\begin{cases} -\frac{\gamma\pi}{6\sqrt{a}} \ge -\mathrm{arccot}(\sinh 2\theta) > -\pi & \text{for}\quad \gamma > 0\ , \\ -\frac{\gamma\pi}{6\sqrt{a}} \le \mathrm{arccot}(-\sinh 2\theta) < \pi & \text{for}\quad \gamma < 0\ . \end{cases} \tag{2.95}$$

On the other hand, we find $|\frac{\gamma}{\sqrt{a}}| > 2\sqrt{21} > 6$. Thus the curvature singularity at $\cot\gamma\rho = -\sinh 2\theta$ is unavoidable.

Class III.ii

The solution is

$$\begin{aligned} ds^2 &= e^{\frac{2}{3}\Phi}\left\{e^{\alpha\rho}\left[-(1+\tilde{Q}\rho)dt^2 - 2\tilde{Q}\rho dt dz_1 + (1-\tilde{Q}\rho)dz_1^2\right] + e^{-2\alpha\rho}dz_2^2\right\} \\ &\quad + e^{-\frac{1}{3}\Phi}(1+\frac{a}{4r^{12}})^{\frac{1}{3}}(dr^2 + r^2 d\Omega_7^2)\,, \\ A_{(3)} &= -\frac{\sinh\gamma\rho\ \sinh 2\theta}{\cosh\gamma\rho - \sinh\gamma\rho\ \cosh 2\theta} dt\wedge dz^1 \wedge dz^2\,, \end{aligned} \tag{2.96}$$

where

$$\begin{aligned} & e^{-\Phi} = \cosh\gamma\rho - \sinh\gamma\rho\ \cosh 2\theta\ , \qquad \cos(6\sqrt{a}\rho) = \frac{1-\frac{a}{4r^{12}}}{1+\frac{a}{4r^{12}}}\ , \\ & -\gamma^2 - 3\alpha^2 = 84a\ . \end{aligned} \tag{2.97}$$

In this class we always have $a \le 0$. Then $(-a/4)^{\frac{1}{12}} < r < \infty$ corresponds to $-\infty < \rho < 0$. In the case $\gamma < 0$, there is a curvature singularity at $\coth\gamma\rho = \cosh 2\theta$, and the range of ρ should be $\gamma^{-1}\text{arccoth}(\cosh 2\theta) < \rho < 0$. In the case $\gamma \ge 0$, the range of ρ should be $-\infty < \rho < 0$, and $r = (-a/4)^{\frac{1}{12}}$ is the singularity in general except for the M2-brane bubbles arising at

$$\alpha = -\tfrac{2}{3}\gamma\ , \qquad \frac{\gamma}{\sqrt{-a}} = 6. \tag{2.98}$$

Class III.iii

The solution is

$$\begin{aligned} ds^2 &= e^{\frac{2}{3}\Phi}\left\{e^{\alpha\rho}\left[-(1+\tilde{Q}\rho)dt^2 - 2\tilde{Q}\rho dt dz_1 + (1-\tilde{Q}\rho)dz_1^2\right] + e^{-2\alpha\rho}dz_2^2\right\} \\ &\quad + e^{-\frac{1}{3}\Phi}(1+\frac{a}{4r^{12}})^{\frac{1}{3}}(dr^2 + r^2 d\Omega_7^2)\,, \\ A_{(3)} &= -\frac{Q\rho}{1-Q\rho} dt\wedge dz^1 \wedge dz^2\,, \end{aligned} \tag{2.99}$$

where

$$e^{-\Phi} = 1 - Q\rho\,, \qquad \cos(6\sqrt{a}\rho) = \frac{1-\frac{a}{4r^{12}}}{1+\frac{a}{4r^{12}}}\,, \qquad -3\alpha^2 = 84a\ . \tag{2.100}$$

In this class we always have $a \le 0$. Then $(-a/4)^{\frac{1}{12}} < r < \infty$ corresponds to $-\infty < \rho < 0$. In the case $Q < 0$, there is a curvature singularity at $Q\rho = 1$, and the range of ρ should be $Q^{-1} < \rho < 0$. In the case $Q \ge 0$, the range of ρ should be $-\infty < \rho < 0$, and $r = (-a/4)^{\frac{1}{12}}$ is the singularity except for the extremal case $\alpha = 0$. In this limit, we have $\rho \sim 1/r^6$, and the solution describes a pp-wave of momentum Q propagating in the worldvolume of a BPS M2-brane.

Class IV.i

The solution is

$$\begin{aligned}
ds^2 &= e^{\frac{2}{3}\Phi}\left[-\left(1+\alpha\rho\cos 2\beta-\tfrac{1}{8}\alpha^2\rho^2\sin^2 2\beta\right)dt^2+\left(1-\alpha\rho\cos 2\beta+\tfrac{1}{8}\alpha^2\rho^2\sin^2 2\beta\right)dz_1^2\right.\\
&\quad\left.-2\left(\alpha\rho\cos 2\beta-\tfrac{1}{8}\alpha^2\rho^2\sin^2 2\beta\right)dtdz_1+\alpha\rho\sin 2\beta dtdz_2+\alpha\rho\sin 2\beta dz_1dz_2+dz_2^2\right]\\
&\quad+e^{-\frac{1}{3}\Phi}(1+\frac{a}{4r^{12}})^{\frac{1}{3}}(dr^2+r^2d\Omega_7^2)\,,\\
A_{(3)} &= \frac{\sin\gamma\rho\cosh 2\theta}{\cos\gamma\rho+\sin\gamma\rho\sinh 2\theta}dt\wedge dz^1\wedge dz^2\,,
\end{aligned} \tag{2.101}$$

where

$$\begin{aligned}
&e^{-\Phi}=\cos\gamma\rho+\sin\gamma\rho\sinh 2\theta\ , \qquad \cos(6\sqrt{a}\rho)=\frac{1-\frac{a}{4r^{12}}}{1+\frac{a}{4r^{12}}}\ ,\\
&\gamma^2=84a\ .
\end{aligned} \tag{2.102}$$

In this case we always have $a>0$. Then $0<r<\infty$ corresponds to $-\pi<6\sqrt{a}\rho<0$, and $r=(a/4)^{\frac{1}{12}}$ corresponds to $6\sqrt{a}\rho=\pi/2$. To avoid the possible curvature singularity at $\cot\gamma\rho=-\sinh 2\theta$, we need

$$\begin{cases} -\frac{\gamma\pi}{6\sqrt{a}}\geq -\mathrm{arccot}(\sinh 2\theta)>-\pi & \text{for}\quad \gamma>0\ ,\\ -\frac{\gamma\pi}{6\sqrt{a}}\leq \mathrm{arccot}(-\sinh 2\theta)<\pi & \text{for}\quad \gamma<0\ . \end{cases} \tag{2.103}$$

On the other hand, we find $|\frac{\gamma}{\sqrt{a}}|=2\sqrt{21}>6$. Thus the curvature singularity at $\cot\gamma\rho=-\sinh 2\theta$ is unavoidable, and there is no regular solution in this class.

Class IV.ii

The solution is

$$\begin{aligned}
ds^2 &= e^{\frac{2}{3}\Phi}\left[-\left(1+\alpha\rho\cos 2\beta-\tfrac{1}{8}\alpha^2\rho^2\sin^2 2\beta\right)dt^2+\left(1-\alpha\rho\cos 2\beta+\tfrac{1}{8}\alpha^2\rho^2\sin^2 2\beta\right)dz_1^2\right.\\
&\quad\left.-2\left(\alpha\rho\cos 2\beta-\tfrac{1}{8}\alpha^2\rho^2\sin^2 2\beta\right)dtdz_1+\alpha\rho\sin 2\beta dtdz_2+\alpha\rho\sin 2\beta dz_1dz_2+dz_2^2\right]\\
&\quad+e^{-\frac{1}{3}\Phi}(1+\frac{a}{4r^{12}})^{\frac{1}{3}}(dr^2+r^2d\Omega_7^2)\,,\\
A_{(3)} &= -\frac{\sinh\gamma\rho\ \sinh 2\theta}{\cosh\gamma\rho-\sinh\gamma\rho\ \cosh 2\theta}dt\wedge dz^1\wedge dz^2
\end{aligned} \tag{2.104}$$

where

$$\begin{aligned}
&e^{-\Phi}=\cosh\gamma\rho-\sinh\gamma\rho\ \cosh 2\theta\ , \qquad \cos(6\sqrt{a}\rho)=\frac{1-\frac{a}{4r^{12}}}{1+\frac{a}{4r^{12}}}\ ,\\
&-\gamma^2=84a\ .
\end{aligned} \tag{2.105}$$

In this class we always have $a\leq 0$. Then $(-a/4)^{\frac{1}{12}}<r<\infty$ corresponds to $-\infty<\rho<0$. In the case $\gamma<0$, there is a curvature singularity at $\coth\gamma\rho=\cosh 2\theta$, and the range of

ρ should be $\gamma^{-1}\text{arccoth}(\cosh 2\theta) < \rho < 0$. In the case $\gamma \geq 0$, the range of ρ should be $-\infty < \rho < 0$, and $r = (-a/4)^{\frac{1}{12}}$ is the singularity except the $\gamma = 0$ limit which is the tachyon wave, obtained in [7].

Class IV.iii

In this class we always have $a = 0$. The solution is

$$\begin{aligned} ds^2 &= e^{\frac{2}{3}\Phi}\Big[-\big(1+\alpha\rho\cos 2\beta - \tfrac{1}{8}\alpha^2\rho^2\sin^2 2\beta\big)\,dt^2 + \big(1-\alpha\rho\cos 2\beta + \tfrac{1}{8}\alpha^2\rho^2\sin^2 2\beta\big)\,dz_1^2 \\ &\quad -2\big(\alpha\rho\cos 2\beta - \tfrac{1}{8}\alpha^2\rho^2\sin^2 2\beta\big)\,dt dz_1 + \alpha\rho\sin 2\beta dt dz_2 + \alpha\rho\sin 2\beta dz_1 dz_2 + dz_2^2\Big] \\ &\quad + e^{-\frac{1}{3}\Phi}(dr^2 + r^2 d\Omega_7^2)\,, \\ A_{(3)} &= -\frac{Q\rho}{1-Q\rho}dt\wedge dz^1\wedge dz^2\,, \end{aligned} \tag{2.106}$$

where

$$e^{-\Phi} = 1 - Q\rho\,, \qquad \rho = -\frac{1}{6r^6}. \tag{2.107}$$

The range $0 < r < \infty$ corresponds to $-\infty < \rho < 0$. In the case $Q < 0$, there is a curvature singularity at $Q\rho = 1$, and the range of ρ should be $Q^{-1} < \rho < 0$. In the case $Q \geq 0$, the range of ρ should be $-\infty < \rho < 0$, and $r = 0$ is nonsingular. This configuration describes a tachyon wave propagating in the M2-brane, obtained in [7].

Table 1: Solutions without a naked singularity

Class	Parameter regimes	Description
I.i	$a > 0,\ \gamma > 0,\ \ -\frac{\pi\gamma}{6\sqrt{a}} \geq -\text{arccot}(\sinh 2\theta)$	Wormhole
I.i	$a > 0,\ \gamma < 0,\ \ -\frac{\pi\gamma}{6\sqrt{a}} \leq \text{arccot}(-\sinh 2\theta)$	Wormhole
I.ii	$a > 0,\ \ -\frac{\pi\gamma}{6\sqrt{a}} \leq \text{arccoth}(\cosh 2\theta)$	Wormhole
I.ii (also in II.ii)	$a < 0,\ \gamma = 6\sqrt{-a},\ \alpha = -4\sqrt{-a},\ \beta = 0$	M2 bubble
I.iii	$a > 0,\ \ -\frac{\pi Q}{6\sqrt{a}} \leq 1$	Wormhole
I.iii (also in II.iii)	$a = \alpha = \beta = 0,\ Q \geq 0$	BPS M2 brane
II.ii	$a < 0,\ \gamma = 6\sqrt{-a},\ \alpha_2 = -4\sqrt{-a},\ \alpha_1 = 8\sqrt{-a}$	Black M2 brane
III.ii	$a < 0,\ \gamma = 6\sqrt{-a},\ \alpha = -4\sqrt{-a}$	M2 bubble with pp-wave
III.iii	$a = \alpha = 0,\ Q \geq 0$	pp-wave on BPS M2 brane
IV.iii	$a = 0,\ Q \geq 0$	Tachyon wave on BPS M2 brane

In summary, we have obtained the most general spherically symmetric M2-branes. We found that there are a total of twelve classes of solutions. All of the solutions without a naked singularity are listed in Table 1.

3 The traversability of wormholes

We have obtained the most general spherically symmetric M2-branes in the previous sections. From Table 1, we can find that the class I.i, I.ii, and I.iii solutions with appropriate parameters describe smooth wormholes that connect two asymptotic regions. We have obtained their ADM masses, linear momenta, as well as the M2 charges measured in each asymptotic region. In this section, we discuss the traversability of these wormholes. The traversability of higher-dimensional Ricci-flat wormholes was discussed in [17].

3.1 Geodesic motion

The metrics of the class I.i, I.ii, and I.iii solutions have the same form. The Lagrangian for geodesic motion is given by

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \\ &= \tfrac{1}{2} e^{-\frac{1}{3}\Phi}\left(1+\frac{a}{4r^{12}}\right)^{\frac{1}{3}}\left[\dot{r}^2 + r^2(\dot{\theta}_1^2 + \sin^2\theta_1\,\dot{\theta}_2^2 + \cdots + \sin^2\theta_1 \ldots \sin^2\theta_6\,\dot{\theta}_7^2)\right] \\ &\quad + \tfrac{1}{2} e^{\frac{2}{3}\Phi - 2\alpha\rho}\dot{z}_2^2 + \tfrac{1}{2} e^{\frac{2}{3}\Phi+\alpha\rho}\left[\cos\beta\rho\,(\dot{z}_1^2 - \dot{t}^2) + 2\sin\beta\rho\,\dot{t}\dot{z}_1\right]\,. \end{aligned} \tag{3.1}$$

where a dot denotes a derivative with respect to the proper time τ. The conserved quantities are

$$\begin{aligned} J &= e^{-\frac{1}{3}\Phi}\left(1+\frac{a}{4r^{12}}\right)^{\frac{1}{3}} r^2 \sin^2\theta_1 \ldots \sin^2\theta_6\,\dot{\theta}_7\,, \\ E &= e^{\frac{2}{3}\Phi+\alpha\rho}(\cos\beta\rho\,\dot{t} - \sin\beta\rho\,\dot{z})\,, \\ p_1 &= e^{\frac{2}{3}\Phi+\alpha\rho}(\cos\beta\rho\,\dot{z} + \sin\beta\rho\,\dot{t})\,, \\ p_2 &= e^{\frac{2}{3}\Phi-2\alpha\rho}\dot{z}_2\,. \end{aligned} \tag{3.2}$$

By the $SO(8)$ symmetry, we can choose an appropriate coordinate system for any given geodesic such that $\theta_i \equiv \frac{1}{2}\pi$ $(i = 1, 2, \ldots, 6)$. The Lagrangian itself is a constant for geodesic motions, *i.e.*, $\mathcal{L} = -\frac{1}{2}\epsilon$, where $\epsilon = 1, 0, -1$ for the timelike, null and spacelike geodesics respectively. It follows that we have

$$e^{-\frac{1}{3}\Phi}\left(1+\frac{a}{4r^{12}}\right)^{\frac{1}{3}}\dot{r}^2 = -\epsilon - e^{\frac{1}{3}\Phi}\left(1+\frac{a}{4r^{12}}\right)^{-\frac{1}{3}}\frac{J^2}{r^2} - e^{-\frac{2}{3}\Phi+2\alpha\rho}p_2^2 + V\,, \tag{3.3}$$

where the potential is

$$\begin{aligned} V &= e^{-\frac{2}{3}\Phi-\alpha\rho}\left[(E^2 - p_1^2)\cos\beta\rho + 2Ep_1\sin\beta\rho\right] \\ &= e^{-\frac{2}{3}\Phi-\alpha\rho}(E^2+p_1^2)\cos(\beta\rho - \lambda)\,, \end{aligned} \tag{3.4}$$

where

$$\lambda = \arcsin\left(\frac{2Ep_1}{E^2+p_1^2}\right)\,. \tag{3.5}$$

The asymptotic regions W_∞ and W_0 correspond to $\rho = 0$ and $\rho = -\frac{\pi}{6\sqrt{a}}$ respectively.

For the class I.i, we have $\gamma^2 \frac{\pi^2}{36a} < \pi^2$, and thus $\beta^2 \frac{\pi^2}{36a} \geq (84a - \gamma^2)\frac{\pi^2}{36a} > \frac{4\pi^2}{3}$. For the classes I.ii and I.iii, we have $\beta^2 \frac{\pi^2}{36a} \geq 84a \frac{\pi^2}{36a} = \frac{7\pi^2}{3}$. It follows that the range of the angle $\beta\rho$ from W_∞ to W_0 is always larger than π. Therefore V must take some negative values in some region of r, and the wormhole is not traversable for both the timelike and null geodesics.

3.2 Timelike trajectories

We now consider traversable timelike trajectories. The metric of the wormholes can be recast into

$$\begin{aligned} ds^2 =& e^{\frac{2}{3}\Phi+\alpha\rho}(\sin\tfrac{1}{2}\beta\rho\, du + \cos\tfrac{1}{2}\beta\rho\, dv)(\sin\tfrac{1}{2}\beta\rho\, dv - \cos\tfrac{1}{2}\beta\rho\, du) + e^{\frac{2}{3}\Phi-2\alpha\rho} dz_2^2 \\ &+ e^{-\frac{1}{3}\Phi}\left(1+\frac{a}{4r^{12}}\right)^{\frac{1}{3}}\left[dr^2 + r^2\, d\Omega_7^2\right] , \end{aligned} \tag{3.6}$$

where (u, v) are light-cone coordinates, defined by

$$u = t - z_1 \,, \; v = t + z_1 \;. \tag{3.7}$$

To get the timelike trajectory from W_+ to W_-, let us consider

$$\begin{aligned} u &= \eta(\cot\tfrac{1}{2}(\beta\rho+c)+1)^2\cos\tfrac{1}{2}c - \eta(\cot\tfrac{1}{2}(\beta\rho+c)-1)^2\sin\tfrac{1}{2}c + c_1 \,, \\ v &= \eta(\cot\tfrac{1}{2}(\beta\rho+c)-1)^2\cos\tfrac{1}{2}c + \eta(\cot\tfrac{1}{2}(\beta\rho+c)+1)^2\sin\tfrac{1}{2}c + c_2 \,, \end{aligned} \tag{3.8}$$

where η, c, c_1, c_2 are arbitrary constants. We have

$$\begin{aligned} &\sin\tfrac{1}{2}\beta\rho\, u' + \cos\tfrac{1}{2}\beta\rho\, v' = \cos\tfrac{1}{2}\beta\rho\, u' - \sin\tfrac{1}{2}\beta\rho\, v' \\ &= -\frac{\eta\beta}{\sin^3\frac{1}{2}(\beta\rho+c)}\frac{d\rho}{dr} = \frac{\eta\beta}{\sin^3\frac{1}{2}(\beta\rho+c)}\frac{\sin(6\sqrt{a}\rho)}{\sqrt{a}r} \,, \end{aligned} \tag{3.9}$$

where a prime denotes a derivative with respect to r. Then the timelike condition

$$e^{\frac{2}{3}\Phi+\alpha\rho}(\sin\tfrac{1}{2}\beta\rho\, u' + \cos\tfrac{1}{2}\beta\rho\, v')(\sin\tfrac{1}{2}\beta\rho\, v' - \cos\tfrac{1}{2}\beta\rho\, u') + e^{-\frac{1}{3}\Phi}\left(1+\frac{a}{4r^{12}}\right)^{\frac{1}{3}} < 0 \tag{3.10}$$

implies

$$\frac{\eta^2\beta^2\, e^{\Phi+\alpha\rho}}{\sin^6(\frac{1}{2}(\beta\rho+c))\, r^{14}\left(1+\frac{a}{4r^{12}}\right)^{\frac{7}{3}}} > 1 \,. \tag{3.11}$$

Note that

$$\frac{\eta^2\beta^2\, e^{\Phi+\alpha\rho}}{\sin^6(\frac{1}{2}(\beta\rho+c))\, r^{14}\left(1+\frac{a}{4r^{12}}\right)^{\frac{7}{3}}} \geq \frac{\eta^2\beta^2\, e^{\Phi+\alpha\rho}}{r^{14}\left(1+\frac{a}{4r^{12}}\right)^{\frac{7}{3}}} \geq 0 \,, \tag{3.12}$$

and 0 is only possible for $r \to 0, \infty$. From the continuation of the expression of the left-hand side of (3.11), its minimum in the region $r > (a/4)^{1/12}$ must be greater than 0 if it is greater than 0 at W_∞; its minimum in the region $0 < r < (a/4)^{1/12}$ must be also greater than 0 if it is greater than 0 at W_0. The most relevant example for the former case is $c = 0$ where

$$\frac{\eta^2\beta^2\, e^{\Phi+\alpha\rho}}{\sin^6(\frac{1}{2}\beta\rho)\, r^{14}\left(1+\frac{a}{4r^{12}}\right)^{\frac{7}{3}}} \to \infty\,, \qquad \text{as} \qquad r \to \infty, \tag{3.13}$$

while the most relevant example for the later case is $c = \frac{\beta\pi}{6\sqrt{a}}$ where

$$\frac{\eta^2\beta^2\, e^{\Phi+\alpha\rho}}{\sin^6(\frac{1}{2}\beta\rho + \frac{\beta\pi}{12\sqrt{a}})\, r^{14}\left(1+\frac{a}{4r^{12}}\right)^{\frac{7}{3}}} \to \infty\,, \qquad \text{as} \qquad r \to 0\,. \tag{3.14}$$

Thus along the trajectories

$$\mathrm{T}_\infty : \quad \begin{cases} u = \eta_\infty(\cot\frac{1}{2}\beta\rho + 1)^2 + c_1, \\ v = \eta_\infty(\cot\frac{1}{2}\beta\rho - 1)^2 + c_2, \end{cases} \tag{3.15}$$

$$\mathrm{T}_0 : \quad \begin{cases} u = \eta_0(\cot(\frac{1}{2}\beta\rho + \frac{\beta\pi}{12\sqrt{a}}) + 1)^2 \cos\frac{\beta\pi}{12\sqrt{a}} \\ \qquad\qquad -\eta_0(\cot(\frac{1}{2}\beta\rho + \frac{\beta\pi}{12\sqrt{a}}) - 1)^2 \sin\frac{\beta\pi}{12\sqrt{a}} + d_1, \\ v = \eta_0(\cot(\frac{1}{2}\beta\rho + \frac{\beta\pi}{12\sqrt{a}}) - 1)^2 \cos\frac{\beta\pi}{12\sqrt{a}} \\ \qquad\qquad +\eta_0(\cot(\frac{1}{2}\beta\rho + \frac{\beta\pi}{12\sqrt{a}}) + 1)^2 \sin\frac{\beta\pi}{12\sqrt{a}} + d_2. \end{cases} \tag{3.16}$$

with sufficiently large $\eta^2_{\infty,0}$, one could travel from $W_{\infty,0}$ to the throat of wormhole $r = (a/4)^{\frac{1}{12}}$, respectively. If the two trajectories could be matched in $r = (a/4)^{\frac{1}{12}}$ smoothly, then one could travel from W_∞ to W_0.

For the trajectories (3.15) and (3.16) matching at $r = (a/4)^{1/12}$, we need

$$\begin{aligned} c_1 &= d_1 + \left(\cot(\tfrac{\beta\pi}{24\sqrt{a}}) + 1\right)^2 \eta_0 \cos\tfrac{\beta\pi}{12\sqrt{a}} - \left(\cot(\tfrac{\beta\pi}{24\sqrt{a}}) - 1\right)^2 \left(\eta_0 \sin\tfrac{\beta\pi}{12\sqrt{a}} + \eta_\infty\right)\,, \\ c_2 &= d_2 + \left(\cot(\tfrac{\beta\pi}{24\sqrt{a}}) - 1\right)^2 \eta_0 \cos\tfrac{\beta\pi}{12\sqrt{a}} + \left(\cot(\tfrac{\beta\pi}{24\sqrt{a}}) + 1\right)^2 \left(\eta_0 \sin\tfrac{\beta\pi}{12\sqrt{a}} - \eta_\infty\right)\,. \end{aligned} \tag{3.17}$$

For the trajectories matching smoothly, we further require both $\sin\frac{1}{2}\beta\rho\, u' + \cos\frac{1}{2}\beta\rho\, v'$ and $\cos\frac{1}{2}\beta\rho\, u' - \sin\frac{1}{2}\beta\rho\, v'$ to match at $r = (a/4)^{1/12}$. From (3.9) we know that it is satisfied when

$$\eta_\infty = -\eta_0. \tag{3.18}$$

In conclusion, our example for timelike trajectories from W_∞ to W_0 is

$$u = \begin{cases} -\eta(\cot\frac{1}{2}\beta\rho + 1)^2 + \eta\left(\cot(\frac{\beta\pi}{24\sqrt{a}}) + 1\right)^2 \cos\frac{\beta\pi}{12\sqrt{a}} \\ \qquad -\eta\left(\cot(\frac{\beta\pi}{24\sqrt{a}}) - 1\right)^2 \left(\sin\frac{\beta\pi}{12\sqrt{a}} - 1\right) + d_1\,, \quad (r > (\frac{1}{4}a)^{\frac{1}{12}})\,; \\ \eta(\cot(\frac{1}{2}\beta\rho + \frac{\beta\pi}{12\sqrt{a}}) + 1)^2 \cos\frac{\beta\pi}{12\sqrt{a}} \\ \qquad -\eta(\cot(\frac{1}{2}\beta\rho + \frac{\beta\pi}{12\sqrt{a}}) - 1)^2 \sin\frac{\beta\pi}{12\sqrt{a}} + d_1\,, \quad (r < (\frac{1}{4}a)^{\frac{1}{12}})\,; \end{cases}$$

$$v = \begin{cases} -\eta(\cot\frac{1}{2}\beta\rho - 1)^2 + \eta\left(\cot(\frac{\beta\pi}{24\sqrt{a}}) - 1\right)^2 \cos\frac{\beta\pi}{12\sqrt{a}} \\ \qquad\qquad +\eta\left(\cot(\frac{\beta\pi}{24\sqrt{a}}) + 1\right)^2 \left(\sin\frac{\beta\pi}{12\sqrt{a}} + 1\right) + d_2\,, \quad (r > (\frac{1}{4}a)^{\frac{1}{12}})\,; \\ \eta(\cot(\frac{1}{2}\beta\rho + \frac{\beta\pi}{12\sqrt{a}}) - 1)^2 \cos\frac{\beta\pi}{12\sqrt{a}} \\ \qquad\qquad +\eta(\cot(\frac{1}{2}\beta\rho + \frac{\beta\pi}{12\sqrt{a}}) + 1)^2 \sin\frac{\beta\pi}{12\sqrt{a}} + d_2\,, \quad (r < (\frac{1}{4}a)^{\frac{1}{12}})\,, \end{cases} \tag{3.19}$$

where η is large enough.

The Lagrangian for this trajectory

$$\begin{aligned} 2\mathcal{L} &= e^{\frac{2}{3}\Phi+\alpha\rho}(\sin\tfrac{1}{2}\beta\rho\,\dot{u} + \cos\tfrac{1}{2}\beta\rho\,\dot{v})(\sin\tfrac{1}{2}\beta\rho\,\dot{v} - \cos\tfrac{1}{2}\beta\rho\,\dot{u}) + e^{-\frac{1}{3}\Phi}\left(1 + \frac{a}{4r^{12}}\right)^{\frac{1}{3}} \dot{r}^2 \\ &= -1 \end{aligned} \tag{3.20}$$

implies

$$\dot{r} = \begin{cases} -\left(\frac{\eta^2\beta^2\, e^{\frac{2}{3}\Phi+\alpha\rho}}{\sin^6(\frac{1}{2}\beta\rho)\, r^{14}\left(1+\frac{a}{4r^{12}}\right)^2} - e^{-\frac{1}{3}\Phi}\left(1 + \frac{a}{4r^{12}}\right)^{\frac{1}{3}}\right)^{-\frac{1}{2}}\,, \\ \qquad\qquad (r > (\frac{1}{4}a)^{\frac{1}{12}})\,; \\ -\left(\frac{\eta^2\beta^2\, e^{\frac{2}{3}\Phi+\alpha\rho}}{\sin^6(\frac{1}{2}\beta\rho+\frac{\beta\pi}{12\sqrt{a}})\, r^{14}\left(1+\frac{a}{4r^{12}}\right)^2} - e^{-\frac{1}{3}\Phi}\left(1 + \frac{a}{4r^{12}}\right)^{\frac{1}{3}}\right)^{-\frac{1}{2}}\,, \\ \qquad\qquad (r < (\frac{1}{4}a)^{\frac{1}{12}})\,. \end{cases} \tag{3.21}$$

The radial velocity is zero at infinity and finite everywhere. Since the velocity is analytic except $r = (a/4)^{1/12}$ and continuous everywhere, the acceleration $\ddot{r}$ should be finite everywhere and would not be continuous only at $r = (a/4)^{1/12}$. As a check, let us consider the simple case which is in class I.i with $\gamma = \alpha = 0$. In this case, we have

$$\begin{aligned} \ddot{r} = -\frac{1}{2}\dot{r}^4 \Bigg[&\frac{\eta^2\beta^2}{\sin^6(\frac{1}{2}\beta\rho + c)\, r^{14}\left(1 + \frac{a}{4r^{12}}\right)^2} \left(\frac{6a}{r^{13}}\left(1 + \frac{a}{4r^{12}}\right)^{-1} - \frac{14}{r}\right. \\ &\left. + \frac{3\beta\sin 6\sqrt{a}\rho}{\sqrt{a}r}\cot(\tfrac{1}{2}\beta\rho + c)\right) + \frac{a}{r^{13}}\left(1 + \frac{a}{4r^{12}}\right)^{-\frac{2}{3}}\Bigg]\,, \end{aligned} \tag{3.22}$$

where

$$c = \begin{cases} 0\,, & (r > (\frac{1}{4}a)^{\frac{1}{12}})\,; \\ \frac{\beta\pi}{12\sqrt{a}}\,, & (r < (\frac{1}{4}a)^{\frac{1}{12}})\,. \end{cases} \tag{3.23}$$

Note that $\rho = -\frac{\pi}{12\sqrt{a}}$ at $r = (a/4)^{1/12}$, and thus $\ddot{r}$ has a jump at this point coming from the $\cot(\frac{1}{2}\beta\rho + c)$ term. At any other point, the expression of $\ddot{r}$ is regular.

The proper acceleration is given by

$$A^\mu = \dot{U}^\mu + \Gamma^\mu{}_{\nu\rho} U^\nu\, U^\rho\,, \tag{3.24}$$

where $U^\mu = \dot{x}^\mu$. Its explicit expression is rather complicated. However, we can choose a coordinate system where the relevant $\Gamma^u{}_{\nu\rho}$ are finite everywhere while U^μ is analytic except $r = (a/4)^{1/12}$ and continues everywhere. Therefore the proper acceleration should also be finite everywhere and not continuous only at $r = 0$. In conclusion, we have obtained a timelike trajectory from W_∞ to W_0 which is regular everywhere except $r = (a/4)^{1/12}$ where the acceleration would have a jump. Such a trajectory is physically acceptable.

4 General spherically symmetric type IIB strings

The $\frac{SL(3,\mathbb{R})}{SO(1,2)} \times \frac{SL(2,\mathbb{R})}{SO(1,1)}$ scalar coset, which we used to construct the most general spherically symmetric M2-branes, can also arise from the dimension reduction of type IIB supergravity on $\mathbb{R}^{1,1}$. The bosonic type IIB action in the Einstein frame is given by [23, 24]

$$\begin{aligned} S_{\rm IIB} = \int d^{10}x\sqrt{-G}\Big(R - \tfrac12(\partial\phi)^2 - \tfrac12 e^{2\phi}(\partial\chi_0)^2 - \tfrac{1}{12}e^{-\phi}H_{(3)}^2 - \tfrac{1}{12}e^{\phi}\hat{F}_{(3)}^2 \\ - \tfrac{1}{480}\hat{F}_{(5)}^2\Big) - \tfrac12\int C_{(4)}\wedge H_{(3)}\wedge F_{(3)}\,, \end{aligned} \tag{4.1}$$

where $H_{(3)} = dB_{(2)}$, $F_{(3)} = dC_{(2)}$, $\hat{F}_{(3)} = F_{(3)} - \chi_0 \wedge H_{(3)}$ and $\hat{F}_{(5)} = dC_{(4)} - \frac12 C_{(2)}\wedge H_{(3)} + \frac12 B_{(2)}\wedge F_{(3)}$. In addition, a self-duality condition $\hat{F}_{(5)} = *\hat{F}_{(5)}$ should be imposed. Since we consider the reduction to eight dimensions and collect the scalars only, the self-dual 5-form field strength plays no role in our discussion.

The reduction ansatz for the metric and the two 2-form tensors is given by

$$\begin{aligned} ds_{10}^2 &= e^{-\frac{1}{2\sqrt3}\tilde\Phi}ds_8^2 + e^{\frac{\sqrt3}{2}\tilde\Phi}dz^{\rm T}\tilde{\mathcal{M}}\,dz \\ &= e^{-\frac{1}{2\sqrt3}\tilde\Phi}ds_8^2 + e^{\frac{\sqrt3}{2}\tilde\Phi}\left(e^{-\Phi}(dz+\chi dt)^2 - e^{\Phi}dt^2\right)\,, \\ C_{(2)} &= \chi_1 dt\wedge dz\,, \qquad B_{(2)} = \chi_2 dt\wedge dz\,. \end{aligned} \tag{4.2}$$

The scalars χ and Φ, associated with $\tilde{\mathcal{M}}$, form a complex scalar that is an $\frac{SL(2,\mathbb{R})}{SO(1,1)}$ coset. The axionic scalars χ_0, χ_1 and χ_2, the dilaton ϕ and the breathing mode $\tilde\Phi$ form the $\frac{SL(3,\mathbb{R})}{SO(1,2)}$ scalar coset with

$$\mathcal{M} = \begin{pmatrix} -e^{\frac{2}{\sqrt3}\tilde\Phi} + e^{\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_1^2 + e^{-\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_2^2 & e^{\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_1 & e^{\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_0\chi_1 - e^{-\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_2 \\ e^{\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_1 & e^{\phi-\frac{1}{\sqrt3}\tilde\Phi} & e^{\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_0 \\ e^{\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_0\chi_1 - e^{-\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_2 & e^{\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_0 & e^{-\phi-\frac{1}{\sqrt3}\tilde\Phi} + e^{\phi-\frac{1}{\sqrt3}\tilde\Phi}\chi_0^2 \end{pmatrix}. \tag{4.3}$$

Comparing with (2.3), we find $\phi = \Phi_1$, $\tilde\Phi = -\Phi_2$, $\chi_0 = \chi_{23}$, $\chi_1 = \chi_{10}$ and $\chi_2 = -\chi_{20}$. These relationships can also be derived from the T duality of the type IIA and type IIB

strings. The Lagrangian for the scalar sector in eight dimensions is given by

$$\begin{aligned}\mathcal{L} &= \sqrt{g}\Big[R_8 + \tfrac{1}{4}\mathrm{tr}(\partial_\mu \mathcal{M}^{-1}\partial^\mu \mathcal{M}) + \tfrac{1}{4}\mathrm{tr}(\partial_\mu \tilde{\mathcal{M}}^{-1}\partial^\mu \tilde{\mathcal{M}})\Big]\\ &= \sqrt{g}\Big[R_8 - \tfrac{1}{2}(\partial\Phi)^2 + \tfrac{1}{2}e^{-2\Phi}(\partial\chi)^2 - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\tilde{\Phi})^2\\ &\qquad - \tfrac{1}{2}e^{2\phi}(\partial\chi_0)^2 + \tfrac{1}{2}e^{\phi-\sqrt{3}\tilde{\Phi}}(\partial\chi_1 + \chi_2\partial\chi_0)^2 + \tfrac{1}{2}e^{-\phi-\sqrt{3}\tilde{\Phi}}(\partial\chi_2)^2\Big]\,. \end{aligned} \tag{4.4}$$

Since the $SL(2,\mathbb{R})$ symmetry associated with $\tilde{\mathcal{M}}$ is part of the general coordinate transformation in type IIB supergravity, it follows that we can put $\tilde{\mathcal{M}}$ in the simplified canonical form. Thus we have the followings:

- **Class i:** $\tilde{\mathcal{C}}$ has a pair of complex eigenvalues,

$$\tilde{\mathcal{M}} = \begin{pmatrix} -\cos\gamma\rho & \sin\gamma\rho \\ \sin\gamma\rho & \cos\gamma\rho \end{pmatrix}\,. \tag{4.5}$$

- **Class ii:** $\tilde{\mathcal{C}}$ has two real eigenvalues, with one timelike and one spacelike eigenvectors,

$$\tilde{\mathcal{M}} = \begin{pmatrix} -e^{\gamma\rho} & 0 \\ 0 & e^{-\gamma\rho} \end{pmatrix}\,. \tag{4.6}$$

- **Class iii:** $\tilde{\mathcal{C}}$ is rank 1 and all of its eigenvalues are zero,

$$\tilde{\mathcal{M}} = \begin{pmatrix} -(1+Q\rho) & -Q\rho \\ -Q\rho & 1-Q\rho \end{pmatrix}\,. \tag{4.7}$$

For the $\frac{SL(3,\mathbb{R})}{SO(1,2)}$ coset, the analysis is much more complicated. We can set $\mathcal{M}(0) = \mathrm{diag}\{-1,1,1\}$ by the rigid rescaling transformation and the rigid gauge transformations. Then we are left with a rigid $SO(1,2)$ residual symmetry. The $SO(2)$ subgroup of $SO(1,2)$, which acts trivially on $\tilde{\Phi}$, comes from the original classical $SL(2,\mathbb{R})$ symmetry of type IIB supergravity.

Given the solution of $\mathcal{M}$ in canonical form as in section 2.2, the general solution can be obtained by performing the residual $SO(1,2)$ transformation $\mathcal{M} \to \Lambda^T\mathcal{M}\Lambda$. The $SO(1,2)$ transformation Λ can be parameterized by the analog of the Euler angles as

$$\begin{aligned}\Lambda &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_1 & -s_1 \\ 0 & s_1 & c_1 \end{pmatrix} \begin{pmatrix} ch & sh & 0 \\ sh & ch & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix}\\ &= \begin{pmatrix} ch & c_2\,sh & -s_2\,sh \\ c_1\,sh & c_1c_2\,ch - s_1s_2 & -c_1s_2\,ch - s_1c_2 \\ s_1\,sh & s_1c_2\,ch + c_1s_2 & -s_1s_2\,ch + c_1c_2 \end{pmatrix}\,, \end{aligned} \tag{4.8}$$

where $c_1 = \cos\theta_1$, $c_2 = \cos\theta_2$, $ch = \cosh\theta_0$, $s_1 = \sin\theta_1$, $s_2 = \sin\theta_2$, and $sh = \sinh\theta_0$.

However, such parametrization gives rise to lengthy and complicated results, so we shall use an alternative parametrization as follows to describe the most general solutions.

Given an arbitrary $\mathcal{C}$ matrix, we can transform it to be the form

$$\mathcal{C} = \begin{pmatrix} \alpha_0 & -\beta_1 & -\beta_2 \\ \beta_1 & \alpha_1 & 0 \\ \beta_2 & 0 & \alpha_2 \end{pmatrix} \tag{4.9}$$

by an $SO(2)$ transformation. Let us suppose that the eigenvalues of $\mathcal{C}$ are $\lambda_\mu(\mu = 0, 1, 2)$ and λ_0 is related to the timelike or null eigenspace. We must have

$$\det(\mathcal{C} - \lambda I_{2p\times 2p}) = \prod_{\mu=0}^{2} (\lambda_\mu - \lambda)\,. \tag{4.10}$$

This implies

$$\beta_i = \pm\sqrt{\frac{\prod_{\mu=0}^{2}(\alpha_i - \lambda_\mu)}{\prod_{j\neq i}(\alpha_i - \alpha_j)}}\,, \qquad i, j = 1, 2. \tag{4.11}$$

If $\mathcal{C}$ belongs to Class I or Class II, we can find a transformation T which transforms (4.9) to the canonical form by solving the three eigenvectors of (4.9). Then performing the inverse transformation T^{-1} on the canonical solution, we arrive at the solution for (4.9) as

$$\mathcal{M} = \Delta^{-1} \begin{pmatrix} \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \alpha_{1,\mu}\alpha_{2,\mu} & \beta_1 \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \alpha_{2,\mu} & \beta_2 \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \alpha_{1,\mu} \\ \beta_1 \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \alpha_{2,\mu} & \beta_1^2 \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \frac{\alpha_{2,\mu}}{\alpha_{1,\mu}} & \beta_1\beta_2 \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \\ \beta_2 \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \alpha_{1,\mu} & \beta_1\beta_2 \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu & \beta_2^2 \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \frac{\alpha_{1,\mu}}{\alpha_{2,\mu}} \end{pmatrix}. \tag{4.12}$$

where

$$\begin{aligned} \Lambda_0 &= \lambda_1 - \lambda_2, \quad \Lambda_1 = \lambda_2 - \lambda_0, \quad \Lambda_2 = \lambda_0 - \lambda_1, \\ \Delta &= \prod_{\mu=0}^{2} \Lambda_\mu, \quad \alpha_{i,\mu} = \alpha_i - \lambda_\mu. \end{aligned} \tag{4.13}$$

An additional $SO(2)$ transformation gives the parametrization of general $\mathcal{C}$ as

$$\mathcal{C} = \begin{pmatrix} \alpha_0 & -(\beta_1\cos\theta - \beta_2\sin\theta) & -(\beta_2\cos\theta + \beta_1\sin\theta) \\ \beta_1\cos\theta - \beta_2\sin\theta & \alpha_1\cos^2\theta + \alpha_2\sin^2\theta & \frac{1}{2}(\alpha_1 - \alpha_2)\sin 2\theta \\ \beta_2\cos\theta + \beta_1\sin\theta & \frac{1}{2}(\alpha_1 - \alpha_2)\sin 2\theta & \alpha_1\sin^2\theta + \alpha_2\cos^2\theta \end{pmatrix}. \tag{4.14}$$

The corresponding solutions in Class I and Class II are

$$\mathcal{M} = \Delta^{-1} \sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \begin{pmatrix} \alpha_{1,\mu}\alpha_{2,\mu} & B_{1,\mu} & B_{2,\mu} \\ B_{1,\mu} & \frac{B_{1,\mu}^2}{\alpha_{1,\mu}\alpha_{2,\mu}} & \frac{B_{1,\mu}B_{2,\mu}}{\alpha_{1,\mu}\alpha_{2,\mu}} \\ B_{2,\mu} & \frac{B_{1,\mu}B_{2,\mu}}{\alpha_{1,\mu}\alpha_{2,\mu}} & \frac{B_{2,\mu}^2}{\alpha_{1,\mu}\alpha_{2,\mu}} \end{pmatrix}, \tag{4.15}$$

where

$$B_{1,\mu} = \beta_1\alpha_{2,\mu}\cos\theta - \beta_2\alpha_{1,\mu}\sin\theta\,, \qquad B_{2,\mu} = \beta_2\alpha_{1,\mu}\cos\theta + \beta_1\alpha_{2,\mu}\sin\theta\,. \tag{4.16}$$

Comparing with (4.3), we can determine the solution of the various fields to be

$$\begin{aligned}
\chi_0 &= \frac{\sum_{\mu=0}^2 e^{\lambda_\mu\rho}\Lambda_\mu \frac{B_{1,\mu}B_{2,\mu}}{\alpha_{1,\mu}\alpha_{2,\mu}}}{\sum_{\mu=0}^2 e^{\lambda_\mu\rho}\Lambda_\mu \frac{B_{1,\mu}^2}{\alpha_{1,\mu}\alpha_{2,\mu}}}, \qquad \chi_1 = \frac{\sum_{\mu=0}^2 e^{\lambda_\mu\rho}\Lambda_\mu B_{1,\mu}}{\sum_{\mu=0}^2 e^{\lambda_\mu\rho}\Lambda_\mu \frac{B_{1,\mu}^2}{\alpha_{1,\mu}\alpha_{2,\mu}}}, \\
e^{\phi-\frac{1}{\sqrt{3}}\tilde{\Phi}} &= \Delta^{-1}\sum_{\mu=0}^{2} e^{\lambda_\mu\rho}\Lambda_\mu \frac{B_{1,\mu}^2}{\alpha_{1,\mu}\alpha_{2,\mu}}, \\
e^{-\frac{2}{\sqrt{3}}\tilde{\Phi}} &= \Delta^{-2}\sum_{\mu>\nu} e^{(\lambda_\mu+\lambda_\nu)\rho}\Lambda_\mu\Lambda_\nu \frac{\beta_1^2\beta_2^2(\alpha_{2,\mu}\alpha_{1,\nu}-\alpha_{1,\mu}\alpha_{2,\nu})^2}{\alpha_{1,\mu}\alpha_{2,\mu}\alpha_{1,\nu}\alpha_{2,\nu}}, \\
\chi_2 &= \frac{\sum_{\mu>\nu} e^{(\lambda_\mu+\lambda_\nu)\rho}\Lambda_\mu\Lambda_\nu \frac{\beta_1^2\beta_2^2(\alpha_{2,\mu}\alpha_{1,\nu}-\alpha_{1,\mu}\alpha_{2,\nu})(B_{1,\nu}\alpha_{1,\mu}\alpha_{2,\mu}-B_{1,\mu}\alpha_{1,\nu}\alpha_{2,\nu})}{\alpha_{1,\mu}\alpha_{2,\mu}\alpha_{1,\nu}\alpha_{2,\nu}}}{\sum_{\mu>\nu} e^{(\lambda_\mu+\lambda_\nu)\rho}\Lambda_\mu\Lambda_\nu \frac{\beta_1^2\beta_2^2(\alpha_{2,\mu}\alpha_{1,\nu}-\alpha_{1,\mu}\alpha_{2,\nu})^2}{\alpha_{1,\mu}\alpha_{2,\mu}\alpha_{1,\nu}\alpha_{2,\nu}}}.
\end{aligned} \tag{4.17}$$

These expressions are still complicated and we shall not discuss further about the properties of the solutions.

If the matrix $\mathcal{C}$ belongs to Class III, the twofold eigenvalues $\lambda_0 = \lambda_1 = \lambda$ have only one eigenvector which is null. We can solve the eigenvector v_2 for λ_2. Based on v_2, we can formulate an orthogonal and normalized basis that transforms $\mathcal{M}$ into the canonical form. Performing the inverse transformation on the canonical solution, we arrive at the solution for (4.9) as

$$\begin{aligned}
\mathcal{M} = -\Lambda^{-2}\Bigg[& e^{\lambda\rho}\begin{pmatrix} -C_0+\alpha_{1,1}\alpha_{2,1}\Lambda\rho & \beta_1(-\alpha_{2,2}+\alpha_{2,1}\Lambda\rho) & \beta_2(-\alpha_{1,2}+\alpha_{1,1}\Lambda\rho) \\ \beta_1(-\alpha_{2,2}+\alpha_{2,1}\Lambda\rho) & \beta_1^2\frac{-C_1+\alpha_{1,1}\alpha_{2,1}\Lambda\rho}{\alpha_{1,1}^2} & \beta_1\beta_2(-1+\Lambda\rho) \\ \beta_2(-\alpha_{1,2}+\alpha_{1,1}\Lambda\rho) & \beta_1\beta_2(-1+\Lambda\rho) & \beta_2^2\frac{-C_2+\alpha_{1,1}\alpha_{2,1}\Lambda\rho}{\alpha_{2,1}^2} \end{pmatrix} \\
& + e^{\lambda_2\rho}\begin{pmatrix} \alpha_{1,2}\alpha_{2,2} & \beta_1\alpha_{2,2} & \beta_2\alpha_{1,2} \\ \beta_1\alpha_{2,2} & \beta_1^2\frac{\alpha_{2,2}}{\alpha_{1,2}} & \beta_1\beta_2 \\ \beta_2\alpha_{1,2} & \beta_1\beta_2 & \beta_2^2\frac{\alpha_{1,2}}{\alpha_{2,2}} \end{pmatrix}\Bigg],
\end{aligned} \tag{4.18}$$

where

$$\begin{aligned}
&\Lambda = \lambda-\lambda_2\,, \qquad \alpha_{i,j} = \alpha_i - \lambda_j, \\
&C_0 = \alpha_{1,1}\alpha_{2,1} + \alpha_{1,1}\Lambda + \alpha_{2,1}\Lambda, \\
&C_1 = \alpha_{1,1}\alpha_{2,1} + \alpha_{1,1}\Lambda - \alpha_{2,1}\Lambda, \\
&C_2 = \alpha_{1,1}\alpha_{2,1} - \alpha_{1,1}\Lambda + \alpha_{2,1}\Lambda.
\end{aligned} \tag{4.19}$$

This result can also be obtained as the $\lambda_0 - \lambda_1 \to 0$ limit of (4.12). An additional $SO(2)$ transformation gives the parametrization of general $\mathcal{C}$ as (4.14), and the corresponding

solution in Class III is

$$\mathcal{M} = -\Lambda^{-2}\left[e^{\lambda_2\rho}\begin{pmatrix} \alpha_{1,2}\alpha_{2,2} & B_{1,2} & B_{2,2} \\ B_{1,2} & \frac{B_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}} & \frac{B_{1,2}B_{2,2}}{\alpha_{1,2}\alpha_{2,2}} \\ B_{2,2} & \frac{B_{1,2}B_{2,2}}{\alpha_{1,2}\alpha_{2,2}} & \frac{B_{2,2}^2}{\alpha_{1,2}\alpha_{2,2}} \end{pmatrix}\right. \tag{4.20}$$

$$\left. + e^{\lambda\rho}\begin{pmatrix} \Lambda^2 - \alpha_{1,2}\alpha_{2,2} & -B_{1,2} & -B_{2,2} \\ -B_{1,2} & -\Lambda^2 - \frac{B_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}} & -\frac{B_{1,2}B_{2,2}}{\alpha_{1,2}\alpha_{2,2}} \\ -B_{2,2} & -\frac{B_{1,2}B_{2,2}}{\alpha_{1,2}\alpha_{2,2}} & -\Lambda^2 - \frac{B_{2,2}^2}{\alpha_{1,2}\alpha_{2,2}} \end{pmatrix} + e^{\lambda\rho}\Lambda\rho\begin{pmatrix} \alpha_{1,1}\alpha_{2,1} & B_{1,1} & B_{2,1} \\ B_{1,1} & \frac{B_{1,1}^2}{\alpha_{1,1}\alpha_{2,1}} & \frac{B_{1,1}B_{2,1}}{\alpha_{1,1}\alpha_{2,1}} \\ B_{2,1} & \frac{B_{1,1}B_{2,1}}{\alpha_{1,1}\alpha_{2,1}} & \frac{B_{2,1}^2}{\alpha_{1,1}\alpha_{2,1}} \end{pmatrix}\right],$$

where

$$B_{1,i} = \beta_1\alpha_{2,i}\cos\theta - \beta_2\alpha_{1,i}\sin\theta\,, \qquad B_{2,i} = \beta_2\alpha_{1,i}\cos\theta + \beta_1\alpha_{2,i}\sin\theta. \tag{4.21}$$

Comparing with (4.3), we can decide the solution of the various fields as

$$\begin{aligned}
\chi_0 &= \frac{e^{\lambda\rho}\left(\frac{B_{1,2}B_{2,2}}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} - \rho\frac{B_{1,1}B_{2,1}}{\alpha_{1,1}\alpha_{2,1}\Lambda}\right) - e^{\lambda_2\rho}\frac{B_{1,2}B_{2,2}}{\alpha_{1,2}\alpha_{2,2}\Lambda^2}}{e^{\lambda\rho}\left(1 + \frac{B_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} - \rho\frac{B_{1,1}^2}{\alpha_{1,1}\alpha_{2,1}\Lambda}\right) - e^{\lambda_2\rho}\frac{B_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2}}, \\
\chi_1 &= \frac{e^{\lambda\rho}\left(\frac{B_{1,2}}{\Lambda^2} - \rho\frac{B_{1,1}}{\Lambda}\right) - e^{\lambda_2\rho}\frac{B_{1,2}}{\Lambda^2}}{e^{\lambda\rho}\left(1 + \frac{B_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} - \rho\frac{B_{1,1}^2}{\alpha_{1,1}\alpha_{2,1}\Lambda}\right) - e^{\lambda_2\rho}\frac{B_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2}}, \\
e^{\phi-\frac{1}{3}\tilde{\Phi}} &= e^{\lambda\rho}\left(1 + \frac{B_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} - \rho\frac{B_{1,1}^2}{\alpha_{1,1}\alpha_{2,1}\Lambda}\right) - e^{\lambda_2\rho}\frac{B_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2}, \\
e^{-\frac{2}{\sqrt{3}}\tilde{\Phi}} &= e^{(\lambda+\lambda_2)\rho}\left(-\frac{\beta_1^2\alpha_{2,2}^2 + \beta_2^2\alpha_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} + \rho\frac{\beta_1^2\beta_2^2(\alpha_{1,2}\alpha_{2,1} - \alpha_{1,1}\alpha_{2,2})^2}{\alpha_{1,1}\alpha_{1,2}\alpha_{2,1}\alpha_{2,2}\Lambda^3}\right) \\
&\quad + e^{2\lambda\rho}\left(1 + \frac{\beta_1^2\alpha_{2,2}^2 + \beta_2^2\alpha_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2}\right. \\
&\qquad \left. -\rho\frac{\beta_1^2\beta_2^2(\alpha_{1,2}\alpha_{2,1} - \alpha_{1,1}\alpha_{2,2})^2 + \alpha_{1,2}\alpha_{2,2}(\beta_1^2\alpha_{2,1}^2 + \beta_2^2\alpha_{1,1}^2)}{\alpha_{1,1}\alpha_{1,2}\alpha_{2,1}\alpha_{2,2}\Lambda^3}\right), \qquad (4.22) \\
e^{-\frac{2}{\sqrt{3}}\tilde{\Phi}}\chi_2 &= e^{(\lambda+\lambda_2)\rho}\left(\frac{B_{2,2}}{\Lambda^2} - \rho\frac{\beta_1^2\beta_2^2(\alpha_{1,2}\alpha_{2,1} - \alpha_{1,1}\alpha_{2,2})(B_{1,1}\alpha_{1,2}\alpha_{2,2} - B_{1,2}\alpha_{1,1}\alpha_{2,1})}{\alpha_{1,1}\alpha_{1,2}\alpha_{2,1}\alpha_{2,2}\Lambda^3}\right) \\
&\quad + e^{2\lambda\rho}\left[-\frac{B_{2,2}}{\Lambda^2} + \rho\left(\frac{\beta_1^2\beta_2^2(\alpha_{1,2}\alpha_{2,1} - \alpha_{1,1}\alpha_{2,2})(B_{1,1}\alpha_{1,2}\alpha_{2,2} - B_{1,2}\alpha_{1,1}\alpha_{2,1})}{\alpha_{1,1}\alpha_{1,2}\alpha_{2,1}\alpha_{2,2}\Lambda^3} + \frac{B_{1,1}}{\Lambda^2}\right)\right].
\end{aligned}$$

Note that the $SL(2,\mathbb{Z})$ multiplet of type IIB superstrings obtained in [25] is a special case of the class III solutions.

If $\mathcal{C}$ belongs to Class IV, the canonical form given by (2.22) itself is related to the general form (4.9) by an $SO(2)$ transformation. Then the most general solution for $\mathcal{C}$ for this class can be obtained from an arbitrary $SO(2)$ transformation acting on the canonical $\mathcal{C}$ given in

(2.22). Thus we have

$$\mathcal{C} = \alpha \begin{pmatrix} \cos\beta & \frac{1}{4}(3\cos(\beta-\theta)+\cos(\beta+\theta)) & -\frac{1}{4}(3\sin(\beta-\theta)-\sin(\beta+\theta)) \\ -\frac{1}{4}(3\cos(\beta-\theta)+\cos(\beta+\theta)) & -\cos(\beta-\theta)\cos\theta & \frac{1}{2}\sin(\beta-2\theta) \\ \frac{1}{4}(3\sin(\beta-\theta)-\sin(\beta+\theta)) & \frac{1}{2}\sin(\beta-2\theta) & \sin(\beta-\theta)\sin\theta \end{pmatrix}. \tag{4.23}$$

Indeed, when $\theta = \frac{1}{2}\beta$, the matrix takes the form of (4.9). The $\mathcal{M}$ is correspondingly given by

$$\mathcal{M} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{1}{8}\alpha^2\rho^2\sin^2\beta \begin{pmatrix} 1 & \cos\theta & \sin\theta \\ \cos\theta & \cos^2\theta & \frac{1}{2}\sin 2\theta \\ \sin\theta & \frac{1}{2}\sin 2\theta & \sin^2\theta \end{pmatrix} + \alpha\rho\times \tag{4.24}$$

$$\begin{pmatrix} \cos\beta & -\frac{1}{4}(3\cos(\beta-\theta)+\cos(\beta+\theta)) & \frac{1}{4}(3\sin(\beta-\theta)-\sin(\beta+\theta)) \\ -\frac{1}{4}(3\cos(\beta-\theta)+\cos(\beta+\theta)) & -\cos(\beta-\theta)\cos\theta & \frac{1}{2}\sin(\beta-2\theta) \\ \frac{1}{4}(3\sin(\beta-\theta)-\sin(\beta+\theta)) & \frac{1}{2}\sin(\beta-2\theta) & \sin(\beta-\theta)\sin\theta \end{pmatrix}.$$

Comparing with (4.3), we can determine that the scalar fields are given by

$$\begin{aligned} \chi_0 &= \frac{\frac{1}{2}\alpha\rho\sin(\beta-2\theta) + \frac{1}{16}\alpha^2\rho^2\sin^2\beta\sin 2\theta}{1-\alpha\rho\cos(\beta-\theta)\cos\theta + \frac{1}{8}\alpha^2\rho^2\sin^2\beta\cos^2\theta}, \\ \chi_1 &= \frac{-\frac{1}{4}\alpha\rho(3\cos(\beta-\theta)+\cos(\beta+\theta)) + \frac{1}{8}\alpha^2\rho^2\sin^2\beta\cos\theta}{1-\alpha\rho\cos(\beta-\theta)\cos\theta + \frac{1}{8}\alpha^2\rho^2\sin^2\beta\cos^2\theta}, \\ \chi_2 &= -\frac{\frac{1}{4}\alpha\rho(3\sin(\beta-\theta)-\sin(\beta+\theta)) - \frac{1}{8}\alpha^2\rho^2\sin^2\beta\sin\theta}{1-\alpha\rho\cos\beta - \frac{1}{8}\alpha^2\rho^2\sin^2\beta}, \\ e^{\phi-\frac{1}{\sqrt{3}}\tilde{\Phi}} &= 1-\alpha\rho\cos(\beta-\theta)\cos\theta + \frac{1}{8}\alpha^2\rho^2\sin^2\beta\cos^2\theta, \\ e^{-\frac{2}{\sqrt{3}}\tilde{\Phi}} &= 1-\alpha\rho\cos\beta - \frac{1}{8}\alpha^2\rho^2\sin^2\beta. \end{aligned} \tag{4.25}$$

Having obtained the results for both $\tilde{\mathcal{M}}$ and $\mathcal{M}$ for all classes of solutions, it is straightforward to lift the solution back to $D = 10$ using (4.2), giving rise to the most general string solutions with $\mathbb{R}^{1,1} \times SO(8)$ isometries.

5 $D = 5$ pure gravity on $\mathbb{R}^{1,1}$

In this section, we consider five-dimensional pure gravity on $\mathbb{R}^{1,1}$. The resulting $D = 3$ gravity theory is described by an $\frac{SL(3,\mathbb{R})}{SO(1,2)}$ coset. This enables us to find the most gen-

eral Ricci-flat solutions with $R^{1,1} \times SO(3)$ isometry. The Einstein-Hilbert action in five dimensions is given by

$$S_5 = \int d^5x \sqrt{-G}\, R\,. \tag{5.1}$$

Performing the dimension reduction on $\mathbb{R}^{1,1}$, we can take

$$ds_5^2 = e^{-\frac{2}{\sqrt{3}}\Phi} ds_3^2 + e^{\frac{1}{\sqrt{3}}\Phi}\left[-e^{\phi}(dt + \chi dz + \tilde{A}_{(1)})^2 + e^{-\phi}(dz + A_{(1)})^2\right]. \tag{5.2}$$

The Lagrangian becomes

$$\mathcal{L}_3 = \sqrt{g}\Big(R_3 - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\Phi)^2 + \tfrac{1}{2}e^{2\phi}(\partial\chi)^2 + \tfrac{1}{4}e^{\phi+\sqrt{3}\Phi}\tilde{F}_{(2)}^2 - \tfrac{1}{4}e^{-\phi+\sqrt{3}\Phi}F_{(2)}^2\Big)\,, \tag{5.3}$$

where

$$\tilde{F}_{(2)} = d\tilde{A}_{(1)} - d\chi \wedge A_{(1)}\,, \qquad F_{(2)} = dA_{(1)}\,. \tag{5.4}$$

The equations of motion are

$$\begin{aligned}\frac{\delta S}{\delta \tilde{A}_{(1)}} &= -d{*}\left(e^{\phi+\sqrt{3}\Phi}\tilde{F}_{(2)}\right) = 0\,,\\ \frac{\delta S}{\delta A_{(1)}} &= d{*}\left(e^{-\phi+\sqrt{3}\Phi}F_{(2)}\right) - {*}\left(e^{\phi+\sqrt{3}\Phi}\tilde{F}_{(2)}\right)\wedge d\chi\\ &= d\left(e^{-\phi+\sqrt{3}\Phi}{*}F_{(2)} - e^{\phi+\sqrt{3}\Phi}{*}\tilde{F}_{(2)}\wedge\chi\right) = 0\,.\end{aligned} \tag{5.5}$$

Thus we can define the dual field of $\tilde{A}^{(1)}$ and $A^{(1)}$ as follows

$$d\tilde{\varphi} = -e^{\phi+\sqrt{3}\Phi}{*}\tilde{F}_{(2)}, \qquad d\varphi = e^{-\phi+\sqrt{3}\Phi}{*}F_{(2)} - e^{\phi+\sqrt{3}\Phi}{*}\tilde{F}_{(2)}\chi\,. \tag{5.6}$$

Then we have

$$\tilde{F}_{(2)} = {*}{*}\,\tilde{F}_{(2)} = -e^{-\phi-\sqrt{3}\Phi}{*}d\tilde{\varphi}\,,\ \ F_{(2)} = {*}{*}\,F_{(2)} = e^{\phi-\sqrt{3}\Phi}{*}(d\varphi - \chi\, d\tilde{\varphi}). \tag{5.7}$$

The Lagrangian then describe an $\frac{SL(3,R)}{SO(1,2)}$ coset

$$\begin{aligned}\mathcal{L} &= \sqrt{g}\Big[R_3 - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}(\partial\Phi)^2 + \tfrac{1}{2}e^{2\phi}(\partial\chi)^2 - \tfrac{1}{2}e^{-\phi-\sqrt{3}\Phi}(\partial\tilde{\varphi})^2 + \tfrac{1}{2}e^{\phi-\sqrt{3}\Phi}(\partial\varphi - \chi\partial\tilde{\varphi})^2\Big]\\ &= \sqrt{g}\Big[R_3 + \tfrac{1}{4}\mathrm{tr}(\partial_\mu\mathcal{M}^{-1}\partial^\mu\mathcal{M})\Big]\,,\end{aligned} \tag{5.8}$$

where $\mathcal{M}$ is the metric related to the $\frac{SL(3,\mathbb{R})}{SO(1,2)}$ scalar coset

$$\mathcal{M} = \begin{pmatrix} -e^{-\phi+\frac{1}{\sqrt{3}}\Phi} + e^{\phi+\frac{1}{\sqrt{3}}\Phi}\chi^2 + e^{-\frac{2}{\sqrt{3}}\Phi}\varphi^2 & e^{\phi+\frac{1}{\sqrt{3}}\Phi}\chi + e^{-\frac{2}{\sqrt{3}}\Phi}\tilde{\varphi}\varphi & e^{-\frac{2}{\sqrt{3}}\Phi}\varphi \\ e^{\phi+\frac{1}{\sqrt{3}}\Phi}\chi + e^{-\frac{2}{\sqrt{3}}\Phi}\tilde{\varphi}\varphi & e^{\phi+\frac{1}{\sqrt{3}}\Phi} + e^{-\frac{2}{\sqrt{3}}\Phi}\tilde{\varphi}^2 & e^{-\frac{2}{\sqrt{3}}\Phi}\tilde{\varphi} \\ e^{-\frac{2}{\sqrt{3}}\Phi}\varphi & e^{-\frac{2}{\sqrt{3}}\Phi}\tilde{\varphi} & e^{-\frac{2}{\sqrt{3}}\Phi} \end{pmatrix}. \tag{5.9}$$

We shall consider the spherically symmetry solutions. As in section 2, the metric in three dimensions is determined by the Einstein equation in the foliating sphere directions; it is given by

$$ds_3^2 = (1+\tfrac{a}{4r^2})^2(dr^2 + r^2 d\Omega_2^2)\,. \tag{5.10}$$

All of the scalars depend on the radial coordinate r only. The scalar equations of motion can be integrated to satisfy the following first-order equation:

$$\mathcal{M}^{-1}\dot{\mathcal{M}} = \mathcal{C}\,, \tag{5.11}$$

where the constant matrix $\mathcal{C}$ satisfies a constraint from the Einstein equation associated with R_{rr}, given by

$$\mathcal{I} \equiv -\tfrac{1}{2}\mathrm{tr}(\mathcal{C}^2) = 4a\,. \tag{5.12}$$

For this $\frac{SL(3,\mathbb{R})}{SO(1,2)}$ coset, we can set $\mathcal{M}(0) = \mathrm{diag}\{-1,1,1\}$ by a rigid coordinate and gauge transformation. We are then left with a rigid $SO(1,2)$ residual symmetry where there is a $SO(1,1)$ subgroup coming from the rigid coordinate transformation. Thus the nontrivial transformations of the solutions form an $\frac{SO(1,2)}{SO(1,1)}$ coset. Before modding out the trivial $SO(1,1)$ part, the most general solutions for $\mathcal{C}$ and $\mathcal{M}$ take the same form as those obtained in previous section, but with

$$\cos(\sqrt{a}\rho) = \frac{1-\frac{a}{4r^2}}{1+\frac{a}{4r^2}}\,. \tag{5.13}$$

Comparing with (5.9), we can find the solutions of the various fields for different classes.

Class I & II:

$$\begin{aligned}
\tilde\varphi &= \frac{\sum_{\mu=0}^2 e^{\lambda_\mu\rho}\Lambda_\mu\frac{B_{1,\mu}B_{2,\mu}}{\alpha_{1,\mu}\alpha_{2,\mu}}}{\sum_{\mu=0}^2 e^{\lambda_\mu\rho}\Lambda_\mu\frac{B_{2,\mu}^2}{\alpha_{1,\mu}\alpha_{2,\mu}}}, \qquad \varphi = \frac{\sum_{\mu=0}^2 e^{\lambda_\mu\rho}\Lambda_\mu B_{2,\mu}}{\sum_{\mu=0}^2 e^{\lambda_\mu\rho}\Lambda_\mu\frac{B_{2,\mu}^2}{\alpha_{1,\mu}\alpha_{2,\mu}}},\\
e^{-\frac{2}{\sqrt3}\Phi} &= \Delta^{-1}\sum_{\mu=0}^2 e^{\lambda_\mu\rho}\Lambda_\mu\frac{B_{2,\mu}^2}{\alpha_{1,\mu}\alpha_{2,\mu}},\\
e^{\phi-\frac{1}{\sqrt3}\Phi} &= \Delta^{-2}\sum_{\mu>\nu} e^{(\lambda_\mu+\lambda_\nu)\rho}\Lambda_\mu\Lambda_\nu\frac{\beta_1^2\beta_2^2(\alpha_{2,\mu}\alpha_{1,\nu}-\alpha_{1,\mu}\alpha_{2,\nu})^2}{\alpha_{1,\mu}\alpha_{2,\mu}\alpha_{1,\nu}\alpha_{2,\nu}},\\
\chi &= \frac{\sum_{\mu>\nu} e^{(\lambda_\mu+\lambda_\nu)\rho}\Lambda_\mu\Lambda_\nu\frac{\beta_1^2\beta_2^2(\alpha_{2,\mu}\alpha_{1,\nu}-\alpha_{1,\mu}\alpha_{2,\nu})(B_{2,\nu}\alpha_{1,\mu}\alpha_{2,\mu}-B_{2,\mu}\alpha_{1,\nu}\alpha_{2,\nu})}{\alpha_{1,\mu}\alpha_{2,\mu}\alpha_{1,\nu}\alpha_{2,\nu}}}{\sum_{\mu>\nu} e^{(\lambda_\mu+\lambda_\nu)\rho}\Lambda_\mu\Lambda_\nu\frac{\beta_1^2\beta_2^2(\alpha_{2,\mu}\alpha_{1,\nu}-\alpha_{1,\mu}\alpha_{2,\nu})^2}{\alpha_{1,\mu}\alpha_{2,\mu}\alpha_{1,\nu}\alpha_{2,\nu}}}\,.
\end{aligned} \tag{5.14}$$

Class III:

$$\tilde\varphi = \frac{e^{\lambda\rho}(\frac{B_{1,2}B_{2,2}}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} - \rho\frac{B_{1,1}B_{2,1}}{\alpha_{1,1}\alpha_{2,1}\Lambda}) - e^{\lambda_2\rho}\frac{B_{1,2}B_{2,2}}{\alpha_{1,2}\alpha_{2,2}\Lambda^2}}{e^{\lambda\rho}(1+\frac{B_{2,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} - \rho\frac{B_{2,1}^2}{\alpha_{1,1}\alpha_{2,1}\Lambda}) - e^{\lambda_2\rho}\frac{B_{2,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2}},$$

$$
\begin{aligned}
\varphi &= \frac{e^{\lambda\rho}(\frac{B_{2,2}}{\Lambda^2} - \rho\frac{B_{2,1}}{\Lambda}) - e^{\lambda_2\rho}\frac{B_{2,2}}{\Lambda^2}}{e^{\lambda\rho}(1+\frac{B_{2,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} - \rho\frac{B_{2,1}^2}{\alpha_{1,1}\alpha_{2,1}\Lambda}) - e^{\lambda_2\rho}\frac{B_{2,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2}},\\
e^{-\frac{2}{\sqrt{3}}\Phi} &= e^{\lambda\rho}(1+\frac{B_{2,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} - \rho\frac{B_{2,1}^2}{\alpha_{1,1}\alpha_{2,1}\Lambda}) - e^{\lambda_2\rho}\frac{B_{2,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2},\\
e^{\phi-\frac{1}{\sqrt{3}}\Phi} &= e^{(\lambda+\lambda_2)\rho}\left(-\frac{\beta_1^2\alpha_{2,2}^2+\beta_2^2\alpha_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} + \rho\frac{\beta_1^2\beta_2^2(\alpha_{1,2}\alpha_{2,1}-\alpha_{1,1}\alpha_{2,2})^2}{\alpha_{1,1}\alpha_{1,2}\alpha_{2,1}\alpha_{2,2}\Lambda^3}\right)\\
&+e^{2\lambda\rho}\left(1+\frac{\beta_1^2\alpha_{2,2}^2+\beta_2^2\alpha_{1,2}^2}{\alpha_{1,2}\alpha_{2,2}\Lambda^2} - \rho\frac{\beta_1^2\beta_2^2(\alpha_{1,2}\alpha_{2,1}-\alpha_{1,1}\alpha_{2,2})^2+\alpha_{1,2}\alpha_{2,2}(\beta_1^2\alpha_{2,1}^2+\beta_2^2\alpha_{1,1}^2)}{\alpha_{1,1}\alpha_{1,2}\alpha_{2,1}\alpha_{2,2}\Lambda^3}\right),\\
e^{\phi-\frac{1}{\sqrt{3}}\Phi}\chi &= e^{(\lambda+\lambda_2)\rho}\left(-\frac{B_{1,2}}{\Lambda^2} - \rho\frac{\beta_1^2\beta_2^2(\alpha_{1,2}\alpha_{2,1}-\alpha_{1,1}\alpha_{2,2})(B_{2,1}\alpha_{1,2}\alpha_{2,2}-B_{2,2}\alpha_{1,1}\alpha_{2,1})}{\alpha_{1,1}\alpha_{1,2}\alpha_{2,1}\alpha_{2,2}\Lambda^3}\right)\\
&+e^{2\lambda\rho}\left[\frac{B_{1,2}}{\Lambda^2} + \rho\left(\frac{\beta_1^2\beta_2^2(\alpha_{1,2}\alpha_{2,1}-\alpha_{1,1}\alpha_{2,2})(B_{2,1}\alpha_{1,2}\alpha_{2,2}-B_{2,2}\alpha_{1,1}\alpha_{2,1})}{\alpha_{1,1}\alpha_{1,2}\alpha_{2,1}\alpha_{2,2}\Lambda^3} - \frac{B_{1,1}}{\Lambda^2}\right)\right].
\end{aligned}
\tag{5.15}
$$

Class IV:

$$
\begin{aligned}
\tilde{\varphi} &= \frac{\frac{1}{2}\alpha\rho\sin(\beta-2\theta)+\frac{1}{16}\alpha^2\rho^2\sin^2\beta\sin2\theta}{1+\alpha\rho\sin(\beta-\theta)\sin\theta+\frac{1}{8}\alpha^2\rho^2\sin^2\beta\sin^2\theta},\\
\varphi &= \frac{\frac{1}{4}\alpha\rho(3\sin(\beta-\theta)-\sin(\beta+\theta))+\frac{1}{8}\alpha^2\rho^2\sin^2\beta\sin\theta}{1+\alpha\rho\sin(\beta-\theta)\sin\theta+\frac{1}{8}\alpha^2\rho^2\sin^2\beta\sin^2\theta},\\
e^{-\frac{2}{\sqrt{3}}\Phi} &= 1+\alpha\rho\sin(\beta-\theta)\sin\theta+\tfrac{1}{8}\alpha^2\rho^2\sin^2\beta\sin^2\theta,\\
e^{\phi-\frac{1}{\sqrt{3}}\Phi} &= 1-\alpha\rho\cos\beta-\tfrac{1}{8}\alpha^2\rho^2\sin^2\beta,\\
\chi &= -\frac{\frac{1}{4}\alpha\rho(3\cos(\beta-\theta)+\cos(\beta+\theta))+\frac{1}{8}\alpha^2\rho^2\sin^2\beta\cos\theta}{1-\alpha\rho\cos\beta-\frac{1}{8}\alpha^2\rho^2\sin^2\beta}.
\end{aligned}
\tag{5.16}
$$

To write the 5D metric explicitly, we further need to convert the solution of φ and $\tilde{\varphi}$ to gauge potentials. In fact, the gauge potentials $A_{(1)}$ and $\tilde{A}_{(1)}$ can be decided without knowing the explicit form of φ and $\tilde{\varphi}$. Note that the components of $\mathcal{M}^{-1}\dot{\mathcal{M}} = \mathcal{C}$

$$
\begin{aligned}
&e^{\phi-\sqrt{3}\Phi}(-\dot{\varphi}+\chi\dot{\tilde{\varphi}}) = \mathcal{C}_{02},\\
&e^{-\phi-\sqrt{3}\Phi}\dot{\tilde{\varphi}}+\chi e^{\phi-\sqrt{3}\Phi}(\dot{\varphi}-\chi\dot{\tilde{\varphi}}) = \mathcal{C}_{12},
\end{aligned}
\tag{5.17}
$$

imply

$$
\begin{aligned}
&dA_{(1)} = F_{(2)} = e^{\phi-\sqrt{3}\Phi} * (\dot{\varphi}-\chi\dot{\tilde{\varphi}})dr = -\mathcal{C}_{02}d\Omega_2 \quad ,\\
&d\tilde{A}_{(1)} - d\chi\wedge A_{(1)} - \chi dA_{(1)} = \tilde{F}_{(2)} - \chi F_{(2)}\\
&= -e^{-\phi-\sqrt{3}\Phi} * \dot{\tilde{\varphi}}dr - \chi e^{\phi-\sqrt{3}\Phi} * (\dot{\varphi}-\chi\dot{\tilde{\varphi}})dr = -\mathcal{C}_{12}d\Omega_2 \quad .
\end{aligned}
\tag{5.18}
$$

Then we can determine the gauge potential via $\mathcal{C}$ directly

$$
\begin{aligned}
&A_{(1)} = \mathcal{C}_{02}\cos\theta_1 d\theta_2 \quad ,\\
&\tilde{A}_{(1)} = \mathcal{C}_{12}\cos\theta_1 d\theta_2 + \chi A_{(1)} = (\mathcal{C}_{12}+\chi\mathcal{C}_{02})\cos\theta_1 d\theta_2 \quad ,
\end{aligned}
\tag{5.19}
$$

where θ_1 and θ_2 are the angular coordinates for the S^2 metric $d\Omega_2^2$.

Finally, let us consider how to mod out the trivial $SO(1,1)$. The $SO(1,1)$ symmetry allows us to set the two constant and thus the $U(1)$ potentials in the following canonical form:

$$\begin{aligned} a)\ \ & \mathcal{C}_{02} = 0; \\ b)\ \ & \mathcal{C}_{12} = 0; \\ c)\ \ & \mathcal{C}_{02} = \mathcal{C}_{12} = 1. \end{aligned} \tag{5.20}$$

Applying this condition to the solutions obtained previously, we then arrive with a full classification of the solutions with the trivial $SO(1,1)$ being modded out.

6 Conclusions

In this paper we obtain the most general spherically symmetric M2-brane and type IIB string solutions. We make use of the fact that any such p-brane solution reduced on the world volume give rise to an instanton solution supported by a scalar coset in the lower-dimensional theory. The Kaluza-Klein reduction of eleven-dimensional supergravity on $\mathbb{R}^{1,2}$ or type IIB supergravity on $\mathbb{R}^{1,1}$ gives rise to a scalar coset $\frac{SL(3,\mathbb{R})}{SO(1,2)} \times \frac{SL(2,\mathbb{R})}{SO(1,1)}$. Using the classifications of $GL(n,\mathbb{R})$ instantons for the Ricci-flat examples [7], we obtain the most general spherically symmetric instanton solutions. Lifting these solutions back to $D = 11$ gives rise to M2-branes.

We find that there are a total twelve classes of M2-branes, including the previously known extremal or nonextremal M2-branes and new smooth M2-brane wormholes that connect two asymptotic regions: one is flat and the other can be either flat or $\mathrm{AdS}_4 \times S^7$. We discuss their properties in some detail. We also obtained the most general spherically symmetric type IIB string solutions by lifting the instanton solutions. Owing to the fact that the $SL(3,\mathbb{R})$ and $SL(2,\mathbb{R})$ factors of the global symmetries are the general coordinate transformation invariance in M theory and type IIB supergravity respectively, the two types of lifting are very different, since we would like to mod out solutions that are related by general coordinate transformation. We also obtain the most general Ricci-flat solutions in five dimensions with $\mathbb{R}^{1,1} \times SO(3)$ isometries.

As was discussed in [26], analytical continuation of p-brane solutions can lead to cosmological solutions [27, 28] where the coordinate r is analytically continued to become the time coordinate. These solutions can be interpreted as spacelike branes [29]. Thus

a proper analytical continuation of our solution will yield the most general S-branes with $\mathbb{R}^{p+1} \times SO(D-p-1)$ isometries,

Our procedure can be generalized to obtain the most general solutions with $\mathbb{R}^{1,p} \times SO(D-p-1)$ isometries in any supergravity in D dimensions. It amounts to a group theoretic study of a certain scalar coset G/H and then classifying the inequivalent classes of the constant matrix $\mathcal{C}$ defined in (2.14). The examples discussed so far all belong to $GL(n,\mathbb{R})$ or $SL(n,\mathbb{R})$, for which the discussion is relatively easy. The situation becomes much more complicated when G belongs to exceptional groups, which can arise in lower-dimensional maximal supergravities. It is of great interest to obtain all of the most general spherically symmetric p-branes in all supergravities.

Acknowledgement

Z.L.W. acknowledges the support by grants from the Chinese Academy of Sciences, a grant from 973 Program with grant No: 2007CB815401 and grants from the NSF of China with Grant No:10588503 and 10535060.

References

[1] M.J. Duff, R.R. Khuri and J.X. Lu, *String solitons,* Phys. Rept. **259**, 213 (1995) [arXiv:hep-th/9412184].

[2] H. Lü and C.N. Pope, *p-brane Solitons in Maximal Supergravities,* Nucl. Phys. B **465**, 127 (1996) [arXiv:hep-th/9512012].

[3] A.A. Tseytlin, *Harmonic superpositions of M-branes,* Nucl. Phys. B **475**, 149 (1996) [arXiv:hep-th/9604035].

[4] I.R. Klebanov and A.A. Tseytlin, *Intersecting M-branes as four-dimensional black holes,* Nucl. Phys. B **475**, 179 (1996) [arXiv:hep-th/9604166].

[5] H. Lü, C.N. Pope, T.A. Tran and K.W. Xu, *Classification of p-branes, NUTs, waves and intersections,* Nucl. Phys. B **511**, 98 (1998) [arXiv:hep-th/9708055].

[6] K. Behrndt, E. Bergshoeff and B. Janssen, *Intersecting D–Branes in Ten and Six Dimensions,* Phys. Rev. D **55**, 3785 (1997) [arXiv:hep-th/9604168].

[7] Z.L. Wang, J. Mei and H. Lü, $GL(n,\mathbb{R})$ *Wormholes and Waves in Diverse Dimensions,* Class. Quant. Grav. **26**, 085020 (2009) [arXiv:0901.0003 [hep-th]].

[8] E. Bergshoeff, W. Chemissany, A. Ploegh, M. Trigiante and T. Van Riet, *Generating Geodesic Flows and Supergravity Solutions,* Nucl. Phys. B **812**, 343 (2009) [arXiv:0806.2310 [hep-th]].

[9] W. Chemissany, J. Rosseel, M. Trigiante and T. Van Riet, *The full integration of black hole solutions to symmetric supergravity theories,* [arXiv:0903.2777 [hep-th]].

[10] E. Cremmer, I.V. Lavrinenko, H. Lü, C.N. Pope, K.S. Stelle and T.A. Tran, *Euclidean-signature supergravities, dualities and instantons,* Nucl. Phys. B **534**, 40 (1998) [arXiv:hep-th/9803259].

[11] C.M. Hull and B. Julia, *Duality and moduli spaces for time-like reductions,* Nucl. Phys. B **534**, 250 (1998) [arXiv:hep-th/9803239].

[12] M.J. Duff and K.S. Stelle, *Multi-membrane solutions of $D = 11$ supergravity,* Phys. Lett. B **253**, 113 (1991).

[13] M.J. Duff, H. Lü and C.N. Pope, *The black branes of M-theory,* Phys. Lett. B **382**, 73 (1996) [arXiv:hep-th/9604052].

[14] M. Cvetič and A.A. Tseytlin, *Non-extreme black holes from non-extreme intersecting M-branes,* Nucl. Phys. B **478**, 181 (1996) [arXiv:hep-th/9606033].

[15] A. Chodos and S. Detweiler, *Spherically Symmetric Solutions In Five-Dimensional General Relativity,* Gen. Rel. Grav. **14**, 879 (1982).

[16] M. Azreg-Ainou and G. Clement, *The Geodesics of the Kaluza-Klein wormhole soliton,* Gen. Rel. Grav. **22**, 1119 (1990).

[17] H. Lü and J. Mei, *Ricci-Flat and Charged Wormholes in Five Dimensions,* Phys. Lett. B **666**, 511 (2008) [arXiv:0806.3111 [hep-th]].

[18] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions,* Phys. Lett. B **76**, 409 (1978).

[19] H. Lü, C.N. Pope and K.W. Xu, *Liouville and Toda Solutions of M-theory,* Mod. Phys. Lett. A **11**, 1785 (1996) [arXiv:hep-th/9604058].

[20] J.X. Lu and S. Roy, *Static, Non-SUSY p-branes in Diverse Dimensions,* JHEP **0502**, 001 (2005) [arXiv:hep-th/0408242].

[21] G. T. Horowitz, *Tachyon condensation and black strings,* JHEP **0508**, 091 (2005) [arXiv:hep-th/0506166].

[22] A. Bergman, H. Lü, J. Mei and C.N. Pope, *AdS Wormholes,* Nucl. Phys. B **810**, 300 (2009) [arXiv:0808.2481 [hep-th]].

[23] J.H. Schwarz, *Covariant Field Equations Of Chiral N=2 D=10 Supergravity,* Nucl. Phys. B **226**, 269 (1983).

[24] E. Bergshoeff, C.M. Hull and T. Ortin, *Duality in the type II superstring effective action,* Nucl. Phys. B **451**, 547 (1995) [arXiv:hep-th/9504081].

[25] J.H. Schwarz, *An SL(2,Z) Multiplet of Type IIB Superstrings,* Phys. Lett. B **360**, 13 (1995) [arXiv:hep-th/9508143].

[26] H. Lü, S. Mukherji and C.N. Pope, *From p-branes to cosmology,* Int. J. Mod. Phys. A **14**, 4121 (1999) [arXiv:hep-th/9612224].

[27] H. Lü, S. Mukherji, C.N. Pope and K.W. Xu, *Cosmological solutions in string theories,* Phys. Rev. D **55**, 7926 (1997) [arXiv:hep-th/9610107].

[28] A. Lukas, B.A. Ovrut and D. Waldram, *Cosmological solutions of type II string theory,* Phys. Lett. B **393**, 65 (1997) [arXiv:hep-th/9608195].

[29] M. Gutperle and A. Strominger, *Spacelike branes,* JHEP **0204**, 018 (2002) [arXiv:hep-th/0202210].